Full Coherent Frequency Conversion between Two Propagating Microwave Modes

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We demonstrate full frequency conversion in the microwave domain using a Josephson three-wave mixing device pumped at the difference between the frequencies of its fundamental eigenmodes. By measuring the signal output as a function of the intensity and phase of the three input signal, idler, and pump tones, we show that the device functions as a controllable three-wave beam splitter or combiner for propagating microwave modes at the single-photon level, in accordance with theory. Losses at the full conversion point are found to be less than $10^{-3}$. Potential applications of the device include quantum information transduction and realization of an ultrasensitive interferometer with controllable feedback.

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A quantum information transducer capable of converting the frequency of a quantum signal without introducing noise is one of the desirable modules in quantum communication [1,2]. With such a device, one could teleport quantum superpositions of ground and excited states of qubits from one system to another with a different transition frequency, without loss of coherence. If a quantum signal can be routed through different frequency channels, optimization of quantum calculation and communication can be advantageously separated.

The simplest scheme for performing frequency conversion is pumping a dispersive medium with nonlinearity $\chi_2$ at precisely the frequency difference between the input and output frequencies [3–10]. Such nonlinear process known as parametric frequency conversion holds the promise of converting the frequency in a unitary, therefore noiseless, manner, namely, without adding loss or dephasing to the processed signal [11]. In the optical domain, although the intrinsic conversion process can be unitary [12–15], technical issues such as filtering the converted signal from noise photons generated via Raman processes and residual pump photons can limit the practical end-to-end conversion efficiency. In the microwave domain, nonlinear elements used in mixers such as Schottky diodes are by contrast intrinsically dissipative and inevitably lead to unacceptable conversion losses. This raises the question as to whether a unitary full conversion of an input signal can be realized practically.

Here, we show that such noiseless full conversion is possible in the microwave domain by operating a dissipationless three-wave mixing element known as the Josephson parametric converter (JPC) [16–18], in a regime of conversion without photon gain. The JPC performs frequency conversion from 8 to 15 GHz with losses below 0.05 dB at full conversion, as opposed to a typical loss of 6 dB in microwave mixers. We reveal the unitary nature of the JPC conversion by utilizing wave interference between the three incommensurate frequencies intervening in the device. The coherence of the conversion process is thus verified without the challenging calibration of the transmission between different input and output lines of disparate frequencies.

To introduce the requirements obeyed by the properties of unitary frequency conversion, we compare in Fig. 1 a

FIG. 1 (color online). In a three-wave coherent converter, the scattered beams have different frequencies which we indicate using different colors: violet, green, and red. Panel (a) depicts a nonlinear dispersive medium operated as a full frequency converter for the incident violet and green beams. Panel (b) depicts a splitter modified into a perfect mirror which constitutes the two-wave analogue of the full frequency converter. Panels (c) and (e) introduce the relationship between frequency conversion and three-wave interference. By operating the device shown in panel (a) as a three-wave 50/50 beam combiner, one can coherently interfere two equal photon flux beams (green and violet) with disparate frequencies via frequency conversion through the third beam (red). Conversely, in a two-wave coherent beam splitter or combiner, complete interference can take place [panels (d) and (f)] when two equal intensity beams with the same frequency are incident at the beam splitter.
three-wave coherent converter and a two-wave coherent beam splitter. Figure 1(a) depicts the desired case of unitary full frequency conversion of an incident propagating beam, i.e., a violet (high frequency) to a green (low frequency) beam, obtained by pumping with adequate strength a dispersive $\chi_2$ medium with a red beam whose frequency is precisely the frequency difference between the violet and green. In Fig. 1(b) we depict the two-wave analogue of such a device; that is, the total reflector whose input and output beams have the same frequency and magnitude but propagate in different directions. Figures 1(c) and 1(e) depict the same three-wave coherent converter shown in Fig. 1(a), now operated at the 50/50 beam-splitting point. At this working point, the device transmits half of the power of the input beams (green and violet) and converts the frequency of the other half, using the third red (pump) beam. Hence, Figs. 1(c) and 1(e) describe destructive interference (DI) scenarios for the two equal photon flux input beams, resulting in the suppression of the green and violet beams, respectively. They also illustrate (1) for a lossless DI to take place between two frequencies, the power must be entirely converted to a third frequency, and (2) the output beams of such a device depend on the phases of all incident coherent beams. For further clarification of the role of the relative phases, we also show in Figs. 1(d) and 1(f) the two-wave analogues of the destructive interference scenarios shown in Figs. 1(c) and 1(e).

Akin to the conceptual three-wave converter depicted in Fig. 1, the JPC is a nondegenerate device with spatial and temporal separation between the idler and signal modes. Its input and output fields share, however, the same spatial port as shown in Fig. 2(a) and must therefore be separated using a circulator. The idler and signal modes of the JPC are differential modes of the microstrip resonators of the device which intersect at a Josephson ring modulator (JRM) [17], as shown in Fig. 2(b). The third mode supported by the device is a nonresonant common-mode drive (pump), whose frequency $f_P$ is set to either the sum of the idler $f_I$ and signal $f_S$ frequencies or their difference. In the case where the pump frequency satisfies $f_P = f_I + f_S$, the device serves as a quantum-limited amplifier which can be used to read out the state of a solid state qubit in real-time [19,20]. In the present case where the pump frequency verifies $f_P = f_I - f_S$ ($f_I > f_S$), the device operates in frequency conversion mode with no photon gain. In this mode, as opposed to amplification [16], the device is not required, according to Caves’ theorem [21], to add any noise. Note that this working regime has been recently the subject of several works in the areas of telecommunication and quantum information processing in optics [4–6,22–24]. Also, in the recent work done at NIST [8,9], the authors parametrically converted photons of different frequencies inside a microwave resonator coupled to a dc-SQUID. Phonon-photon parametric frequency conversion is also at play in the active cooling of a micromechanical or nanomechanical resonator mode [1,25,26]. In our work, by contrast, it is photons from two different spatial and temporal modes that are interconverted.

When operated in conversion mode and under the stiff pump condition, the JPC can be described as an effective two-port beam splitter whose scattering parameters can be adjusted by varying the pump tone of the device. In this mode, part of the incoming wave at the idler (signal) port is transmitted to the signal (idler) port after being down-converted (up-converted) via emission (absorption) of pump photons, while the remaining part is reflected off the idler (signal) port.

In Fig. 3(a) we display measurements of the reflection parameter $|r|^2$ at the signal port (filled red circles) and the idler-to-signal conversion parameter $|r'|^2$ (filled blue circles) as a function of the normalized applied pump power $P_P/P_{P_0}$, where $P_{P_0}$ is the pump power at which $|r'|^2$ is minimum. In the reflection measurement, a coherent tone at $f_S = 8.402$ GHz is applied to the signal port with input power $P_{S_0} = -123$ dBm, which corresponds to about 0.1 photons on average in the signal resonator, while in the conversion measurement an equivalent coherent tone at $f_I = 14.687$ GHz is applied to the idler port [as shown in Fig. 3(c)]. The frequency of the applied pump tone is $f_P = 6.285$ GHz. Both measurements are taken using a spectrum analyzer centered at $f_S$ in zero frequency span mode. The output power measured as a function of $P_P/P_{P_0}$ is normalized relative to the reflected signal power obtained with no applied pump power. There, the JPC has unity reflection,

![Image of circuit diagram](image.png)
a slight shift in power between the peak of the conversion and the minimum of the reflection data. The solid blue and red curves drawn using the equations listed in the caption of Fig. 3 correspond to theory expressions for the conversion and reflection parameters, respectively [27]. In Fig. 3(b) we used filled red and blue circles to plot the dimensionless pump power parameters $|\rho|^2$ and $|\rho'|^2$, which are used to fit the reflection and conversion data as a function of $P_P/P_{P0}$. The solid black line satisfying the relation $|\rho|^2 = P_P/P_{P0}$ corresponds to the expected dependence of $|\rho|^2$ on $P_P/P_{P0}$ for the ideal JPC model with stiff pump approximation. The solid red and blue curves are polynomial fits to the $|\rho|^2$ and $|\rho'|^2$ parameters. The fits satisfy three important properties: (1) they pass through the origin ($|\rho'|^2 = 0$ for $P_P = 0$); (2) they have a leading linear term in the limit of small pump powers; (3) the fits for $|\rho|^2$ and $|\rho'|^2$ coincide for $P_P/P_{P0} < 1$. They therefore show that the parameter $|\rho|^2$ is in fact a renormalized pump strength, at least for $P_P/P_{P0} < 1$.

In addition to the device dependence on pump power shown in Fig. 3, the relative pump phase plays an important role as well. The down-converted idler and the up-converted signal acquire a phase shift which depends on the pump phase. We utilize this phase dependence as shown in Fig. 4 in order to sensitively interfere signal and idler beams. In this measurement, we applied two coherent tones to the signal and idler ports having the same frequencies and powers as in the measurement of Fig. 3. We have also phase-locked the three independent coherent waves to the 10 MHz reference oscillator of a rubidium atomic clock. The color mesh in Fig. 4 depicts the interference fringes of the reflected and converted signals generated at the signal port as a function of $P_P/P_{P0}$ and the relative pump phase. As can be seen in which can be measured within $\pm 0.5$ dB accuracy due to a finite impedance mismatch in the output line.

As can be seen in Fig. 3, the splitting ratio between the converted and reflected portions of the beam is set by the device parameters and the pump amplitude. By varying the intensity of the pump, the splitting ratio of the device can be changed continuously from zero conversion and total reflection, to complete conversion and perfect absorption. It is important to mention that the input power applied to the idler port in the conversion measurement is set to yield, at the 50/50 beam-splitting working point, the same signal output power as the one measured in reflection for $P_{P0}$. It is remarkable that with this single calibration of input powers, which balances the output at the 50/50 beam-splitting point, the device satisfies quite well the equation of conservation of total photon number $|r|^2 + |t|^2 = 1$ ($|r| = |t|$) in the range $P_P/P_{P0} < 1$ (see the cyan circles). When the critical power is traversed $P_P/P_{P0} \geq 1$, we observe a progressive breaking of unitarity $|r|^2 + |t'|^2 < 1$. This can be explained by increased nonlinear effects at elevated pump powers resulting in frequency conversion to higher modes of the system. However, we find that in the vicinity of $P_{P0}$, the conversion loss defined as the deviation of $|r|^2 + |t'|^2$ from unity is less than $\pm 10^{-2}$ within $\pm 2.5\%$ accuracy. We also observe
the figure, the interference contrast is low for $P_p \ll P_{p0}$ and $P_p/P_{p0} \rightarrow 1$ which correspond to almost total reflection and conversion, respectively. A maximum interference contrast is obtained, on the other hand, for $P_p/P_{p0}$ close to 0.32. We interpret this point as that where our device functions as a 50/50 beam splitter or combiner. This is further shown in Fig. 5, where we display a cross section of the interference fringes shown in Fig. 4 at the 50/50 beam-splitting working point. The maximum and minimum points of the curve correspond to constructive and destructive interference conditions, respectively. The other lines represent different reference measurements taken under the same experimental conditions as explained in the text. The input beams applied in each measurement are given in the table and described in the device schematics on the right.

FIG. 5 (color online). The blue curve is a cross section of the wave interference modulation curve shown in blue is plotted as a function of the relative phase of the pump. The red line represents the reflected signal power at the beam combiner in the MZI setup, while the idler serves as a reference, the information is carried by the phase of the signal for incoming idler and signal waves. Assuming, for example, the information is carried by the phase of the signal in the MZI setup, while the idler serves as a reference, the information can be decoded from the incoming signal by measuring the generated interference at the signal port. Moreover, the set point of the device, which yields maximum phase sensitivity, can be maintained in situ by compensating the shift using the pump phase.

Discussions with R. J. Schoelkopf and P. T. Rakich are gratefully acknowledged. The assistance of M. Power in compensating the shift using the pump phase. In addition to quantum information transduction, the device has several potential useful applications, such as cooling of a readout cavity of a qubit by swapping the “hot” cavity photons, for instance, at 8 GHz, by “cold” reservoir photons at 15 GHz, and realization of a Mach-Zehnder interferometer (MZI) scheme for microwaves with real-time feedback. In the latter scheme, the device would function as an interferometric 50/50 beam combiner for incoming idler and signal waves. Assuming, for example, the information is carried by the phase of the signal in the MZI setup, while the idler serves as a reference, the information can be decoded from the incoming signal by measuring the generated interference at the signal port. Moreover, the set point of the device, which yields maximum phase sensitivity, can be maintained in situ by compensating the shift using the pump phase.

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