Hot non-equilibrium quasiparticles in transmon qubits

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Non-equilibrium quasiparticle excitations degrade the performance of a variety of superconducting circuits. Understanding the energy distribution of these quasiparticles will yield insight into their generation mechanisms, the limitations they impose on superconducting devices, and how to efficiently mitigate quasiparticle-induced qubit decoherence. To probe this energy distribution, we directly correlate qubit transitions with charge-parity switches in an offset-charge-sensitive transmon qubit, and find that quasiparticle-induced excitation events are the dominant mechanism behind the residual excited-state population in our samples. The observed quasiparticle distribution would limit T1 to ≈ 200 µs, which indicates that quasiparticle loss in our devices is on equal footing with all other loss mechanisms. Furthermore, the measured rate of quasiparticle-induced excitation events is greater than that of relaxation events, which signifies that the quasiparticles are more energetic than would be predicted from a thermal distribution describing their apparent density.

The adverse effects of non-equilibrium quasiparticles (QPs) ubiquitous in aluminum superconducting devices have been recognized in a wide variety of systems, including Josephson junction (JJ) based superconducting qubits [1–11], kinetic-inductance [12–14] and quantum-capacitance [15] detectors, devices for current metrology [16], Andreev qubits [17–19], and proposed Majorana qubits [20, 21]. While recent efforts to reduce the density of QPs in superconducting qubits have shown some improvement in the relaxation times of devices limited by QP-induced loss [11, 22–24], understanding the energy distribution of non-equilibrium QPs may shed light on their source and further help to mitigate their effects. Furthermore, it has been suggested that “hot” non-equilibrium QPs may be responsible for the residual excited state population seen in superconducting qubits at low temperatures [8, 25, 26], though this has yet to be confirmed directly.

In this letter, we report signatures of hot non-equilibrium QPs observed in the correlations between qubit transitions and QP-tunneling events. An offset-charge sensitive transmon qubit was used to directly detect switches in the charge-parity of the transmon islands associated with individual QPs tunneling across the JJ [9]. We correlated these charge-parity switches with transitions between the ground and first-excited states of the transmon, and found that QP tunneling accounts for ≈ 30% of all qubit relaxation events and ≈ 90% of excitation events. The measured ratio of the QP-induced excitation and relaxation rates is greater than one, defining what we refer to as a “hot” energy distribution of tunneling QPs, which is at odds with a thermal distribution accounting for their estimated density. These results confirm previous suspicions that non-equilibrium QPs are responsible for the residual excited state population in transmon qubits [8, 25, 26], and emphasize the need for further understanding of QP-induced loss.

Ideally, QPs would be in thermal equilibrium with the cryostat (temperature T ≈ 20 mK, for modern dilution refrigerators), and their spontaneous generation would be exponentially suppressed by the superconducting gap ∆ in the density of states (DoS). However, there is consensus that non-equilibrium QPs exist in a variety of superconducting quantum circuits, with an observed fraction of broken Cooper pairs zqp ≈ 10−8–10−6 [1, 3, 4, 11, 23, 25, 27, 29] which is orders of magnitude greater than would be predicted in thermal equilibrium. In a

![FIG. 1: QP-induced transitions in transmon qubits. (a) The DoS ν, in the excitation representation as a function of the reduced energy ε/∆, in the leads of a superconductor-insulator-superconductor (SIS) JJ. Grey arrows represent tunneling processes of QPs (purple). The QPs can gain energy by relaxing the qubit (dashes), lose energy by exciting the qubit (dots), or only change the charge parity (solid). (b) The two lowest energy levels of an offset-charge-sensitive transmon qubit (y-axis not to scale) as a function of offset-charge n, which is written in units of 2e. T0 and T1 are time-averaged energies of the ground and first-excited states, respectively, assuming ergodic fluctuations of n and/or charge parity. Arrows correspond to the processes described in (a) that change the charge parity between odd and even.](image)
transmon \cite{30}. QP tunneling across the JJ will always change the excess charge on the islands by 1e, switching the charge parity of the junction electrodes between “even” and “odd” \cite{2}. Tunneling QPs couple to the phase across the JJ \cite{1 16}, and consequently can induce qubit transitions [Fig. 1]. The values of $\Delta_{QP}^0$ quoted above correspond to a QP bath in equilibrium at a temperature ($\sim 130-190$ mK) higher than $T$. Under this assumption, QP-induced relaxation of the qubit would vastly outweigh QP-induced excitation. As we will show, this is not observed in our devices, indicating that this effective temperature does not adequately describe the QP energy distribution.

To directly probe the interaction between non-equilibrium QPs and a transmon qubit, we slightly relax the transmon-defining condition that the Josephson coupling energy $E_C$ is much greater than the charging energy $E_C$. In this regime, the ground-to-excited-state-transition frequency $f_{01} = (E_1 - E_0)/h$ has a measurable dependence on charge parity, switching between $f_{01} \pm \delta f_{01}$ when a QP tunnels across the JJ [Fig. 1(b)].

The deviation $\delta f_{01}$ is a sinusoidal function of the dimensionless offset-charge $n_g$, which undergoes temporal fluctuations due to reconfiguration of mobile charges in the environment. Because $\hbar \delta f_{01}(n_g) \ll k_B T$, we can assume that QP tunneling dynamics will not depend on $n_g$. The authors of Ref. \cite{9} took advantage of this frequency splitting to track $n_g$, map the charge parity onto the state of a transmon, and correlate qubit relaxation with parity switches\cite{31}. Extending their experiment, we extract not only the QP-induced relaxation rate, but also the QP-induced excitation rate by detailed modeling of the correlations between charge-parity switches and qubit transitions.

The main text will focus on a single transmon qubit with average frequency $f_{01} = 4.400$ GHz and $E_1/E_C \approx 23$, corresponding to a maximum even-odd splitting $2\delta f_{01}(0) = 3.18$ MHz. The average measured relaxation time $T_1 = 95 \mu s$ is on par with state-of-the-art transmons, and the equilibrium ground state population $P_{0}^{eq} = 0.74$ corresponds to an effective qubit temperature of 160 mK. Data from a second sample with similar parameters is discussed in \cite{32}. Chips were mounted in an Al 3D rectangular readout cavity \cite{33} (frequency $f_r = 9.204$ GHz and linewidth $\kappa_r/2\pi = 1.8$ MHz) and cooled in a cryogen-free dilution refrigerator to a base temperature of 20 mK. Devices were measured in the dispersive regime of circuit-QED \cite{34} (dispersive shift $\chi_{QD}/2\pi = 3.8$ MHz) and a Josephson Parametric Converter (JPC) \cite{35} was used to achieve a single-shot qubit-readout fidelity of $\approx 0.97$ in 3.84 $\mu$s with an average readout-resonator occupation $\bar{n} \approx 3$. For more detail regarding the readout signal chain, electromagnetic shielding of the sample, and device fabrication, see \cite{32}.

We first investigated the slow temporal fluctuations of $n_g$ by monitoring $\delta f_{01}(n_g)$ as a function of time using the Ramsey sequence depicted in Fig. 2a. The carrier frequency of the Gaussian $\pi/2$-pulses is chosen to be $f_{QD}$, which is symmetrically detuned from the even and odd charge-parity states at all values of $n_g$. This ensures that the phase evolution of even- and odd-parity states on the equator of the Bloch sphere will interfere constructively, resulting in Ramsey fringes [Fig. 2(b)] characterized by a single oscillation frequency $\delta f_{01}(n_g)$ and a decay constant $T_2$ that is insensitive to fast charge-parity switches. Repeated Ramsey experiments [Fig. 2(c)] show that $n_g$ fluctuates on a timescale of minutes, which is long enough to perform experiments that rely on prior knowledge of $\delta f_{01}(n_g)$.

Using a similar pulse sequence [Fig. 3(a)], we map the charge parity of the transmon onto the qubit state \cite{9}. Two $\pi/2$-pulses, now about orthogonal axes, are separated by a delay $\tau(n_g) = 1/4 \delta f_{01}(n_g)$, resulting in an effective $\pi$-pulse conditioned on charge parity ($\pi_{e,o}$). This charge-parity-mapping operation only discerns between charge-parity states corresponding to transition frequencies greater-than or less-than $f_{QD}$. Without loss of generality, we refer to these as “even” and “odd” charge parity, respectively. The conditioning of ($\pi_{e,o}$) can be changed by varying the relative phases of the constituent $\pi/2$-pulses. The charge parity $P = (2M_1 - 1)(2M_2 - 1)$ is calculated in post-processing. To observe QP-tunneling events in real time, we repeated the charge-parity-mapping sequence every $\Delta t_{exp} = 10 \mu$s for $\sim 600$ ms [Fig. 3(b)]. The power spectral density $S_{PP}$ of these parity fluctuations was averaged over 20 independent charge-parity jump traces [Fig. 3]. $S_{PP}$ was fit to the characteristic form for a random telegraph signal (a modified Lorentzian), from which a parity-switching timescale $T_P = 77 \pm 1 \mu$s and $F = 0.91 \pm 0.01$ were obtained \cite{32}. Because $n_g$ drifts ran-
where $\rho$ is the probability of finding the system in qubit state $i$ and charge parity $\alpha$, and $\bar{\tau}$ is read as “not $i$.” We evolve the above model numerically with initial conditions determined by $M_2$ and $P$ and fit to all eight quantities $\rho(j, pp' | i)(\tau)$, a subset of which are shown in Fig. 4(a), (b).

In addition, we calculate the charge-parity autocorrelation function $\langle PP' \rangle_{ij}(\tau)$, conditioned on the outcomes $m_2 = i$ and $m_3 = j$, respectively [Fig. 4(d)], and fit to functions of the form

$$\langle PP' \rangle_{ij}(\tau) = \rho_{ij}^i(0) \left( \rho_{ij}^j(\tau) - \rho_{ij}^\bar{j}(\tau) \right) + \rho_{ij}^\bar{j}(\tau) + \rho_{ij}^\bar{j}(\tau).$$ (2)

The maximum correlation $\langle PP' \rangle_{ij}(0)$ is limited by the fidelity of the correlation measurement, and qualitatively, the deviation of $\langle PP' \rangle_{ij}(0)$ from this maximum amplitude is related to the ratio $\Gamma_{ij}^\text{eff}/\Gamma_{ij}$ [Fig. 4(d)].

The above analysis does not account for any measurement infidelities, which can skew the observed correlations. These include parity- and qubit-state-dependent errors, such as spontaneous qubit transitions during the parity-mapping sequence, as well as global errors such as pulse infidelity due to uncertainty in $\delta f_{ij}(n_g)$. We stress that proper modeling of these errors is necessary to accurately extract the conditional rates. After these considerations, we fit all eight permutations of $\rho(j, pp' | i)(\tau)$ and the four $\langle PP' \rangle_{ii}(\tau)$ curves simultaneously to the master equation model (lines in Fig. 4) [32]. The slight disagreement at short $\tau$ may be due to measurement-induced qubit transitions, even at low readout power [30, 31].

From our model with measurement errors taken into account, we extract $1/\Gamma_{ij}^\text{eff} = 110 \pm 1 \mu$s, $1/\Gamma_{ii}^\text{eff} = 77 \pm 1 \mu$s, $1/\Gamma_{10}^\text{eff} = 447 \pm 10 \mu$s, $1/\Gamma_{01}^\text{eff} = 400 \pm 7 \mu$s, $1/\Gamma_{10} = 182 \pm 2 \mu$s, and $1/\Gamma_{01} = 6000 \pm 1000 \mu$s. As a check of consistency, we calculate $T_1$, $\Gamma_{ij}^\text{eff}$, and $T_P$ from the eigenvalues and eigenvectors of the master equation, and find that they agree with the quantities quoted above. A second transmon was found to have similar rates [32].

These rates have numerous implications for our understanding of non-equilibrium QPs in our transmon qubits. First, the limit on $T_1$ of this sample imposed by QPs is $\Gamma_{10}^\text{eff} + \Gamma_{01}^\text{eff} = 211 \pm 3 \mu$s, compared to a limit of $\Gamma_{10} + \Gamma_{01} = 177 \pm 2 \mu$s imposed by all other loss mechanisms. This puts QP-induced dissipation on par with the sum of all other dissipation channels, contributing significantly to qubit relaxation $\Gamma_{10}/\Gamma_{10} = 0.29 \pm 0.01$. Second, the ratio $\Gamma_{01}/\Gamma_{10} = 0.94 \pm 0.03$ indicates that QP-induced excitation accounts for the vast majority of the residual transmon excited-state population [Fig. 4(a)], confirming previous suspicions [8, 26]. Finally, $\Gamma_{01}/\Gamma_{10} = 1.12 \pm 0.03$, which is direct evidence of a highly-energetic distribution of QPs. Naïvely applying Fermi-Dirac statistics and detailed balance yields $\Gamma_{01}/\Gamma_{10} = \exp(-h f_{01}/k_B T_{\text{eff}})$, which predicts a negative effective QP temperature $T_{\text{eff}} \approx -2 \text{K}$ in our device. This is evidence that the QP energy distribution is not localized to the gap edge, but has a characteristic energy
various cryostat mixing-chamber temperatures insinuate into the QP energy distribution. This discrepancy may provide some additional insight into the temperature dependence of the quasiparticle bath. (c) Transition rates as extracted from the master equation, in units of \( \tau \). The crossing of curves with \( pp' = -1 \) (black-dashed line) indicates a negative effective temperature of the quasiparticle bath. (d) The charge-parity autocorrelation function \( \langle PP' \rangle \) conditioned on the outcomes \( m_2 = i \) and \( m_3 = j \).

We find that all parity-switching rates \( \Gamma_{ij} \) increase after \( \sim 140 \text{ mK} \), at which point \( T_1, T_P \), and \( \Gamma_{01}^{\text{eo}}/\Gamma_{10}^{\text{eo}} \) all begin to decrease. Modeling the temperature dependence of these rates requires some ansatz about the QP energy distribution, which is typically assumed to be localized near the gap edge \([4, 6]\). In light of our data, this approximation may not be valid for QPs in our system. However, we use this assumption to compare our results with other reports of QP density \( \chi_{\text{qp}} \) in superconducting circuits. If we assume that the populations of non-equilibrium QPs and equilibrium QPs \([6]\) are independent, the total \( \chi_{\text{qp}} \) is the sum:

\[
\chi_{\text{qp}} = \chi_{\text{qp}}^0 + \sqrt{2\pi k_B T / \Delta} e^{-\Delta/k_B T}. \tag{3}
\]

We use \( \Delta = 205 \mu \text{eV} \), consistent with DC measurements of similar films, which accounts for the increase of \( \Delta \) with reduction of Al thickness \([38]\). Under the assumptions in Ref. \([4, 6]\), \( \Gamma_{10}^{\text{eo}} \) should scale linearly with \( \chi_{\text{qp}} \). We see this approximate scaling in our data, with additional structure that may be explained by a temperature-dependent QP-recombination rate [Fig. 5b]. Fitting this greater than \( \Delta + h f_{10} \). Conversely, \( \Gamma_{01}^{\text{eo}}/\Gamma_{10}^{\text{eo}} = 0.03\pm0.01 \), indicating that the non-QP dissipative baths coupled to the transmon are relatively “cold” [Fig. 4(b)], with an effective temperature \( \sim 60 \text{ mK} \). The observation that \( \Gamma_{11}^{\text{eo}} > \Gamma_{00}^{\text{eo}} \) is not yet explained by theoretical predictions \([31]\). This discrepancy may provide some additional insight into the QP energy distribution.

We repeated the correlation measurement [Fig. 4] at various cryostat mixing-chamber temperatures \( T \) [Fig. 5]. We find that all parity-switching rates \( \Gamma_{ij}^{\text{eo}} \) increase after \( \sim 140 \text{ mK} \), at which point \( T_1, T_P \), and \( \Gamma_{01}^{\text{eo}}/\Gamma_{10}^{\text{eo}} \) all begin to decrease. Modeling the temperature dependence of these rates requires some ansatz about the QP energy distribution, which is typically assumed to be localized near the gap edge \([4, 6]\). In light of our data, this approximation may not be valid for QPs in our system. However, we use this assumption to compare our results with other reports of QP density \( \chi_{\text{qp}}^0 \) in superconducting circuits. If we assume that the populations of non-equilibrium QPs and equilibrium QPs \([6]\) are independent, the total \( \chi_{\text{qp}} \) is the sum:

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data yields $x_{qp}^0 \approx 1 \times 10^{-7}$, which agrees with other recent experiments [11, 12, 22, 23]. We note that QPs high above the gap will effect $\Gamma_{qp}$ differently than those near the gap edge, and thus the accuracy of this approximation depends on the non-equilibrium QP energy distribution.

While the generation mechanisms of non-equilibrium QPs are currently not well understood, knowledge of the QP energy distribution may hint at their source. In this letter we have shown that QPs are more energetic than a Fermi-Dirac distribution accounting for their apparent density $x_{qp}^0$ would suggest. Further quantitative analysis of the measured parity switching rates, together with modeling of QP dynamics in our Al films, could reveal the energy range of QP-generating excitations. Proper filtering of RF lines, light-tight shielding [39, 40], and well-thermalized components are now standard ingredients for reducing the QP density, all of which have been taken into account. The Supplemental Material of this letter [42] describes the filtering and shielding of our samples in further detail. One should note that the authors of Ref. [9] reported $T_P$ one order of magnitude greater than what we have presented, with one experimental difference being their use of a Cu readout cavity instead of a superconducting Al cavity. The techniques described above and in Ref. [9] provide a tool to distinguish the effects of these various experimental factors on QP generation.

In conclusion, the correlations between charge-parity switches and qubit transitions in an offset-charge sensitive transmon indicate that QP-induced loss can be responsible for a significant fraction of dissipation in state-of-the-art superconducting qubits. Additionally, we confirm that hot QPs with a highly-excited energy distribution are responsible for the residual excited state population at low temperature in our samples. These devices could act as powerful tools to further probe the energy distribution of non-equilibrium QPs, and could assess the efficacy of QP-reduction techniques, such as induced Abrikosov vortices [11, 22, 23, 29] or galvanically connected QP traps [44, 48].

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[32] See supplemental material.


Supplemental materials for “Hot non-equilibrium quasiparticles in transmon qubits”

SUMMARY OF DEVICES

<table>
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<tr>
<th>Sample</th>
<th>$f_0$ (GHz)</th>
<th>$2f_0$ (MHz)</th>
<th>$T_1$ (µs)</th>
<th>$1/T_{10}^m$ (µs)</th>
<th>$1/T_{11}^m$ (µs)</th>
<th>$1/T_{10}^e$ (µs)</th>
<th>$1/T_{11}^e$ (µs)</th>
<th>$1/T_{01}^m$ (µs)</th>
<th>$1/T_{01}^e$ (µs)</th>
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<td>3.18</td>
<td>95 ± 5</td>
<td>77 ± 1</td>
<td>110 ± 1</td>
<td>77 ± 1</td>
<td>447 ± 10</td>
<td>400 ± 7</td>
<td>182 ± 2</td>
</tr>
<tr>
<td>B</td>
<td>4.255</td>
<td>4.96</td>
<td>44 ± 2</td>
<td>96 ± 1</td>
<td>135 ± 2</td>
<td>92 ± 2</td>
<td>900 ± 100</td>
<td>400 ± 13</td>
<td>61 ± 1</td>
</tr>
</tbody>
</table>

Error estimates on all parameters are extracted from experimental uncertainty and do not reflect slow changes in those quantities over time. Sample A is the device reported in the main text. Sample B was measured in a separate cool-down under nominally identical conditions, though we did not study the temperature dependence of its conditional transition rates.

EXPERIMENTAL SETUP

QP dynamics may be influenced by various aspects of the experimental setup including RF filtering, radiation shielding, use of magnetic materials, and thermalization of the sample. Fig. S1 shows a schematic representation of the RF-lines and shielding inside the cryostat. Magnetic fields can induce vortices which have been shown to decrease QP loss, though this advantage can be undermined by vortex flow dissipation if the magnetic field at the sample is too strong. The transmon was mounted in a separate Cryoperm magnetic shield from the JPC, and special care was taken to not include any strongly magnetic materials inside the shield in order to establish a baseline understanding of QP dynamics in our system. The aluminum sample holder/readout cavity was mounted to a copper bracket using brass screws and molybdenum washers, which were tested prior to use with a magnetometer. A copper plate coated with carbon black suspended in Stycast was placed inside the Cryoperm shield and thermalized to the sample mounting bracket with copper braid. This is an attempt to absorb any photons that leak into the shield. A copper thermalization braid was attached directly to the Al readout cavity, providing a direct thermal link to the mixing chamber stage.

DEVICE FABRICATION

The devices were patterned in a bilayer of Microposit A4 PMMA and EL13 copolymer on a c-plane sapphire substrate by a 100 keV Vistec EBPG 5000+ using standard electron-beam lithography techniques. The JJ mask was designed using the “bridge-free-technique.” The JJ electrodes were formed from 20 and 30 nm thin-film Al, e-beam evaporated in a Plassys UMS300 at an angle of ±20°.

PSD OF CHARGE-PARITY SWITCHES

Repeated measurements of charge-parity produce a parity-jump trace that looks like a symmetric random telegraph signal with variance = 1. The power spectral density of these parity fluctuations, $S_{pp}$, is fit to a modified Lorentzian of the form

$$S_{pp}(f) = \frac{4F^2/T_P}{(2/T_P)^2 + (2\pi f)^2} + (1 - F^2)\Delta t_{exp}$$

Above, $T_P = 76 ± 1$ µs is the characteristic charge-parity switching rate, $F = 0.91 ± 0.01$ is the fidelity of the parity mapping, and $\Delta t_{exp} = 10$ µs is the sampling period of the signal. This model assumes that the detection errors leading to non-unity $F$ are uncorrelated with charge-parity, though $T_1$-errors tend to bias toward measuring even charge parity. A “chi-squared” analysis of the model suggests that this has a negligible effect on the output of the model. For more details, see Ref. [59].
MODELING CORRELATIONS BETWEEN QUBIT TRANSITIONS AND CHARGE-PARITY SWITCHES

We measured correlations between charge-parity switches and qubit transitions, which reveals the extent to which the qubit coherence is limited by non-equilibrium quasiparticle excitations. To correlate these processes, we perform two charge-parity mapping sequences, separated by a variable delay $\tau$ [Fig. S2]. From this we sort our measurement sequences conditioning on starting in qubit state $i$ and parity $p$, and ending up in qubit state $j$ and parity $p'$. We compute two quantities from this data: the conditioned probabilities of all of these events $\rho(j,pp'|i)(\tau)$, and the qubit-state-conditioned charge-parity autocorrelation function $\langle PP'\rangle_{ij}(\tau)$. To model the dynamics between states of the system, we define a master equation describing the dynamics of joint qubit-state and charge-parity occupation probabilities $\rho_{\alpha i}$.

$$\dot{\rho}_{\alpha i} = \left( \Gamma_{\alpha i}^{\alpha} + \Gamma_{i\alpha}^{\alpha} + \Gamma_{i\alpha}^{\alpha} \right) \rho_{\alpha i}\quad + \Gamma_{\alpha i}^{\alpha} \rho_{\alpha i}^{\bar{\alpha}} + \Gamma_{i\alpha}^{\alpha} \rho_{i\alpha}^{\bar{\alpha}} + \Gamma_{i\alpha}^{\alpha} \rho_{i\alpha}^{\bar{\alpha}}. \quad (S2)$$

Here, $\Gamma_{\alpha i}^{\alpha}$ is a conditional transition rate, with $i$ ($\bar{i}$) and $\alpha$ ($\bar{\alpha}$) denoting the conditioned (other) qubit state and charge parity respectively. Because the charge dispersion of the transmon energy levels is small relative to the scale of thermal fluctuations, the conditional rates are symmetric with the exchange of $\alpha$ and $\bar{\alpha}$. We evolve this master equation with initial conditions set by conditioning on the initial qubit and charge-parity state. The full model is solved numerically and fit to measured values of all eight permutations of $\rho(j,pp'|i)(\tau)$ and all four permutations of $\langle PP'\rangle_{ij}(\tau)$, to extract $\Gamma_{00}^{\alpha}, \Gamma_{11}^{\alpha}, \Gamma_{10}^{\alpha}, \Gamma_{01}^{\alpha}$, and $\Gamma_{00}^{\alpha}$.

The measured values of $\rho(j,pp'|i)(\tau)$ and $\langle PP'\rangle_{ij}(\tau)$ are susceptible to various measurement infidelities that are not included in the model above, and we must modify our fit functions to include these infidelities. Single state-discrimination errors will on average decrease $\langle PP'\rangle_{ij}(\tau)$, and $T_1$ errors during the parity mapping will impart an infidelity that depends on both the charge-parity and the qubit state at the start of the parity mapping. Other
measurement inefficiencies are approximately independent of qubit state and charge parity, which contribute to a global fidelity $\mathcal{F}_g$ of the parity-mapping sequence. For example, because $n_g$ varies uncontrollably in time, each sequence of pulse calibrations and parity-autocorrelation measurement must be completed on a timescale faster than a few minutes. Any variation of $n_g$ between the tuning of pulses and the completion of the experiment will introduce qubit-pulse errors, which along with qubit dephasing during $\tau(n_g)$, contribute to $\mathcal{F}_g$. In practice, $\mathcal{F}_g$ is occasionally very low, which we attribute to spontaneous jumps in $n_g$ between the time when $\delta f_{01}(n_g)$ is determined and the correlation measurement. Since we do not know $\mathcal{F}_g$ a priori, we include it in the model as an additional fit parameter, and exclude independent measurement sequences which fall below a threshold $\mathcal{F}_g$. This threshold is 0.5 at low temperatures, where the vast majority of measurements meet this criteria. This threshold must be relaxed at higher fridge temperatures due to increased qubit dephasing.

State-discrimination errors can be sufficiently reduced by ignoring measurement sequences in which any of the four measurements do not meet a stringent state-assignment threshold. We histogram all qubit measurements, fit to a sum of two Gaussian distributions, and exclude measurement sequences where any of the four measurements fall near the half-way point between distributions. In practice, this thresholding removes between 10% and 50% of measurement sequences, depending on the amplitude and integration time of the readout signal, in order to achieve state-discrimination fidelity of greater than 0.9999. The readout amplitude was limited to an average photon number $\bar{n} \approx 3$ to avoid measurement induced qubit transitions [S36, S37].

Each parity-mapping sequence consists of an initial qubit measurement, the Ramsey pulses for parity-mapping, and a final qubit measurement. Because of stringent thresholding, we assume state-assignment with perfect fidelity that is achieved at the midpoint of the readout pulse. There is therefore a time $\tau_1$ between the midpoint of $M_1$ and the beginning of the Ramsey pulses, and time $\tau_2$ between the end of the Ramsey pulses and the midpoint of $M_2$ during which $T_1$ errors can occur [Fig. S2], which we include in our model. Errors during $\tau_1$ and $\tau_2$ from $T_1$ events are included explicitly in the model, and errors between the $\pi/2$-pulses are included implicitly via a global mapping fidelity $\mathcal{F}_g$.

Qubit-state dependent $T_1$ events affect the fidelity with which we determine the charge-parity. For example, let’s say the parity-mapping sequence is chosen such that it enacts a $\pi$-pulse conditioned on being in the even charge-parity state (this is very in the following discussion). If the system is in state $|0\rangle$ (odd), one would expect to measure $m_1 = 0 \rightarrow m_2 = 0$, but $T_1$ errors will appear as $0 \rightarrow 1$ with a probability $\Gamma_{01}(\tau_1 + \tau_2)$. If the system state is $|0\rangle$ (even), one would expect to measure $0 \rightarrow 1$, but $T_1$ errors will appear as $0 \rightarrow 0$ with a probability $\Gamma_{01}\Gamma_{10}(\tau_1 + \tau_2)$. Similar expressions can be found for the system starting in $|1\rangle$. Since there is no physical preference for even or odd parity, we average over parity dependence in the error rates and only consider the probability of starting in an initial state. However, parity-dependent errors will introduce artificial correlations between $P$ and $P'$. To remedy this, we vary whether each parity-mapping sequence performs an effective $\pi$-pulse on the even- or odd-charge-parity state. Assuming near-perfect state discrimination fidelity and equal probability to measure odd or even parity (with balanced pulse conditioning), these errors will only depend on the qubit state at the beginning of the mapping. For the first parity-mapping sequence $P$, we define an error probability

$$\gamma_P^{ij} = (1 - \mathcal{F}_g) + (\mathcal{P}_{0}^{ij}(\tau_{1}\Gamma_{01} + \tau_{2}\Gamma_{10}) + \mathcal{P}_{1}^{ij}(\tau_{1} + \tau_{2})\Gamma_{10})/2 \quad (S3)$$
Above, $P_{ij}^n$ is the probability that $m_1 = n$ at the beginning of $P$ in measurement sequences with qubit-conditioning $m_2 = i$ and $m_3 = j$. Similarly, for the second parity mapping sequence $P'$ we define

$$\gamma_{ij} = (1 - \mathcal{F}_y) + ((\tau_1 + \tau_2)\Gamma_{ij} + (\tau_1\Gamma_{ij} + \gamma_{ij})/2 \quad (S4)$$

This error probability is independent of $i$, and does not have additional qubit-state weighting because we assume near-perfect conditioning of $j$.

Without accounting for any errors, $\rho(j, pp' i)(\tau) = \rho_{ij}(0)\rho_{ij}^p(\tau)$. Errors in the determination of $\rho_{ij}^p(0)$ shuffle the initial probability from the conditioned parity $\rho_{ij}^p(0)$ to the other parity $\rho_{ij}^p(0)$ with a rate $\gamma_{ij}$. We evolve the master equation with these errors accounted for in the initial conditions, in that the conditioned probability $\rho_{ij}^p(0)$ is no longer unity. Then, applying errors in the second parity mapping explicitly, we find:

$$\rho(j, pp' i)(\tau) = (1 - \gamma_{ij})\rho_{ij}^p(\tau) + \gamma_{ij}\rho_{ij}^p(\tau) \quad (S5)$$

We calculate $\langle PP'\rangle_{ij}(\tau)$ directly from these conditional probabilities

$$\langle PP'\rangle_{ij}(\tau) = \frac{\rho(j, +1 i)(\tau) - \rho(j, -1 i)(\tau)}{\rho(j, +1 i)(\tau) + \rho(j, -1 i)(\tau)} \quad (S6)$$

To extract the rates quoted in Table S1, we fit to all eight permutations of $\rho(j, pp' i)(\tau)$ and all four permutations of $\langle PP'\rangle_{ij}$ simultaneously.


[S32] See supplemental material.


