Electrodynamic Dip in the Local Density of States of a Metallic Wire

F. Pierre, H. Pothier, P. Joyez, Norman O. Birge,* D. Esteve, and M. H. Devoret
Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique, Saclay, 91191 Gif-sur-Yvette, France
(Received 15 January 1999; revised manuscript received 3 February 2000)

We have measured the differential conductance of a tunnel junction between a thin metallic wire and a thick ground plane, as a function of the applied voltage. We find that near zero voltage, the differential conductance exhibits a dip, which scales as $1/\sqrt{V}$ down to voltages $V \sim 10k_BT/e$. The precise voltage and temperature dependence of the differential conductance is accounted for by the effect on the tunneling density of states of the macroscopic electrodynamics contribution to electron-electron interaction, and not by the short-ranged screened-Coulomb repulsion at microscopic scales.

DOI: 10.1103/PhysRevLett.86.1590 PACS numbers: 73.23.Hk, 71.55.Jv, 73.40.Gk

Tunneling offers a unique tool to probe interactions between electrons in conducting materials, because the bare electron which tunnels has to be accommodated by the correlated electronic liquid resulting from these interactions. In the past, the effect of interactions on tunneling electrons has been investigated in two different regimes. In the early 1980's Altshuler and Aronov [1] calculated by perturbation theory the effect of the dynamically screened electron-electron interaction in diffusive conductors. They found that this interaction leads to a reduction of the local one-particle density of states (LDOS) near the Fermi energy, and hence of the tunneling conductance at low voltage (the so-called zero-bias anomaly). The LDOS theory successfully explains tunneling experiments performed with extended tunnel junctions, mostly on highly disordered metals and semiconductors [2]. In the more recently addressed "Coulomb blockade" regime, the effect of the long-range part of the electron-electron interaction is so strong that the tunneling conductance can vanish at low voltage. This regime is treated by a phenomenological approach based on macroscopic electromagnetism [3,4] which emphasizes (i) the capacitance C of the tunnel junction formed by the electrodes and the tunnel barrier and (ii) the impedance $Z(\omega)$ of the circuit in which the junction is embedded (the so-called "environmental" impedance). It predicts in particular that the tunneling conductance of a junction will vary at voltages V satisfying $k_BT \ll eV \ll \hbar/RC$ as $[eV/(\hbar/RC)]^{2R/R_K}$, with $R_K = h/e^2$, provided $Z(\omega) = R$ for frequencies $\omega < 1/RC$ (k_BT is the thermal energy). The Coulomb blockade for such a small tunnel junction has been observed in the presence of a large environmental impedance placed in series with the junction, and with relatively weakly disordered metallic electrodes [5,6].

In this Letter, we present an experiment in which the tunneling density of states has been measured for a mesoscopic wire separated from a ground plane by a tunnel oxide barrier. While the experiment remains in the regime where the perturbation theory of Altshuler and Aronov is applicable, the reduction of the density of states at low voltages is entirely controlled, in contrast with previous experiments on extended junctions, by the macroscopic electrodynamic contribution to the electron-electron interaction, as in Coulomb blockade experiments. Our experimental result confirm the link between zero-bias anomalies and Coulomb blockade which has been put forward theoretically [7–9].

According to the real-space expression of the LDOS in the perturbative regime [7], the variation $\delta G = \frac{dI}{dV}(V) - G_t$ of the differential conductance of a tunnel junction of arbitrary shape and dimension between a "dirty" metallic electrode and a perfectly conducting ground plane is given by

$$\frac{\delta G}{G_t} = -\int_{\mathcal{B}} \frac{d^2 r_0}{A_{\mathcal{B}}} \int_0^\infty \frac{d\omega}{\pi\hbar} F_T(eV, \hbar\omega) \times \mathrm{Im}\left[\int d^3r\, d^3r'\, p(r_0, r, \omega) \times U(r, r', \omega) p(r', r_0, \omega)\right], \quad (1)$$

where $\mathcal{B}$ is the interface, with area $A_{\mathcal{B}}$, between the junction oxide and the electrode, $F_T(\varepsilon_1, \varepsilon_2) = \int dE \frac{\partial f_T(E)}{\partial E} \times [f_T(E + \varepsilon_1 + \varepsilon_2) - f_T(E + \varepsilon_1 - \varepsilon_2)]$ is the thermal smearing kernel, f_T being the Fermi function at temperature T, G_t the tunnel conductance, $p(r_0, r, \omega)$ is the time Fourier transform of the probability density that an electron injected at r_0 on $\mathcal{B}$ diffuses in the electrode to point r within a time t, and $U(r, r', \omega)$ is the dynamically screened interaction potential between electrons. We have included here only the largely dominant exchange contribution to the LDOS [10]. The computation of the integral in (1) is greatly simplified when one uses the general relation [8,11,12] $\int d^3r\, p(r_0, r, \omega) U(r, r', \omega) = e^2 z(r_0, r', \omega)$, where $z(r_0, r', \omega) = V(r', \omega)/I(r_0, \omega)$ is the two-point impedance relating the voltage $V(r', \omega)$ at point r' in the electrode to a current $I(r_0, \omega)$ injected at point r_0 (linear electrodynamic response is assumed for the electrodes). Following Nazarov [8,11,13], one can therefore define the effective impedance $Z_{\mathrm{eff}}(r_0, \omega)$ seen by an electron tunneling at r_0 as

$$Z_{\mathrm{eff}}(r_0, \omega) = \int d^3r'\, z(r_0, r', \omega) p(r', r_0, \omega)/\mathcal{N},$$

 0031-9007/01/86(8)/1590(4)$15.00

where $\mathcal{N} = \int d^3r' p(r', r_0, \omega) = 1/i\omega$, and rewrite Eq. (1) in a form identical to the Coulomb blockade prediction in the low environmental impedance limit. In particular, at $T = 0$, $F_T(\varepsilon_1, \varepsilon_2) = \theta(\varepsilon_2 - \varepsilon_1)$ and

$$\frac{\delta G}{G_t} = -2 \int_{\mathcal{B}} \frac{d^2 r_0}{A_{\mathcal{B}}} \int_{eV/\hbar}^{\infty} \frac{\mathrm{Re} Z_{\mathrm{eff}}(r_0, \omega)}{R_K} \frac{d\omega}{\omega}. \quad (2)$$

When $\mathrm{Re}Z_{\mathrm{eff}}$ does not involve the electrons in the tunnel junction itself but only in the external circuit as is usually the case for small tunnel junctions, $\int_{\mathcal{B}} \frac{d^2 r_0}{A_{\mathcal{B}}} Z_{\mathrm{eff}}(r_0, \omega)$ simply reduces to the parallel combination of the environmental impedance $Z(\omega)$ and of the junction capacitance C [3,4].

Our experiment was designed so that the two-point impedance $z(r_0, r, \omega)$ could be simply determined from the dc resistivity of the electrodes, the distributed capacitance of the junction, and the external impedance, while the form of $p(r_0, r, \omega)$ had a negligible influence on $Z_{\mathrm{eff}}(r_0, \omega)$. These conditions are satisfied for electrodes made from a good metal, in which electroneutrality can be assumed for frequencies up to those corresponding to the highest measured voltage. We present the geometry of our sample in Fig. 1(a). The tunnel junction consists of a 29 μm-long, 250 nm-wide, and 22 nm-thick aluminum wire covered by a 190 nm-thick aluminum film. An insulating barrier was grown from the wire by oxidation in pure oxygen at 0.7 bar for 3 h. In order to obtain high-quality junctions, we have used the shadow mask technique which allows us to fabricate the whole sample without exposing it to air [14]. The sample was mounted in a copper box thermally anchored to the mixing chamber of a dilution refrigerator. All electrical connections to the sample were coaxial cables with microwave filters at 4 K and at the temperature of the sample [15]. A magnetic field of 0.4 T was applied perpendicular to the plane of the sample in order to suppress superconductivity in aluminum. The measurements described below were identical in a 0.2 T field, indicating that superconductivity in aluminum had been completely destroyed. The resistance of the wire, measured at 30 mK, was 350 Ω, corresponding to a sheet resistance $R_\square = 3\ \Omega$. The differential conductance of the tunnel junction, measured between the ground plane and either end of the wire, was $G = 26.35\ \mu$s at 30 mK and for $V = 1.5$ mV. The conductance of the junction displays at large scales a quadratic voltage dependence, attributed to the finite barrier height [16]. This quadratic term has an amplitude of several percent for $V = 100$ mV, with a minimum at $V = 15$ mV. At sub-Kelvin temperatures, the conductance of the junction displays a depression at low voltage. Figure 2 shows in a log-log plot the relative change of the differential conductance $\delta G/G_t$ as a function of the applied voltage V. The voltage here is at most 1 mV, so the quadratic term is negligible. In Fig. 3 the rounded cusp observed near zero voltage is presented in a linear plot. The conductance dip fills up and broadens as the temperature is increased. In the inset of Fig. 3, we show the temperature dependence of the conductance at zero voltage. Note that the tunnel conductance G_t is experimentally ill defined at the 0.1% level because the conductance depends on voltage and temperature at large scales, as explained above. In order to compare data with theory, we have taken G_t as a fit parameter (see below), and the experimental data in Figs. 2 and 3 have been normalized to the best-fit value $G_t = 26.392\ \mu$s.

We now discuss the interpretation of our experiment in terms of the preceding reformulated LDOS theory. When

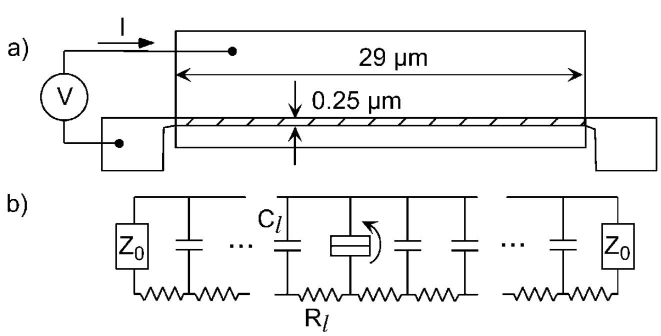

FIG. 1. (a) Schematic of the experiment. A tunnel junction (dashed area) is formed by the overlap of an Al wire and a thick Al counter electrode. A magnetic field suppresses superconductivity. (b) Equivalent circuit entering in the theoretical predictions. The environment of each junction element is two RC transmission lines terminated by an impedance Z_0

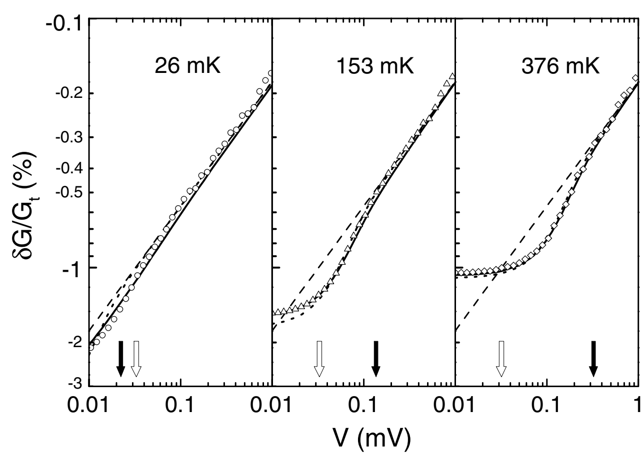

FIG. 2. Symbols: measured variations of the differential conductance of the tunnel junction, normalized to $G_t = 26.392\ \mu$S, as a function of voltage V, for 3 values of the temperature. Each curve corresponds to an average of ten to fifteen voltage sweeps. Solid lines: prediction of the full theory including the effect of temperature and of the finite length of the wire [equivalent circuit shown in Fig. 1(b)]. Dotted lines: predictions for an infinite wire with the same parameters, including the temperature. Dashed lines: predictions for the infinite wire at $T = 0$ showing the $V^{-1/2}$ dependence. Black arrows indicate the position of the crossover voltage $V = 10k_BT/e$. White arrows indicate the energy $\hbar D^*/(L/2)^2$.

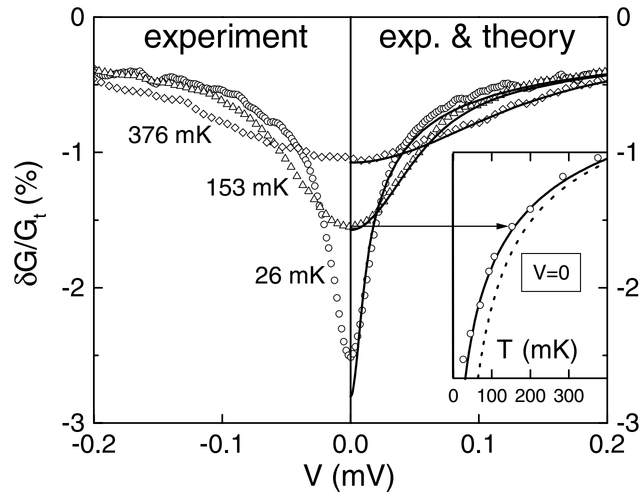

FIG. 3. Symbols in main panel: same experiment as in Fig. 2, but with data near $V = 0$ plotted on linear scale. Solid lines: Predictions for our finite length wire. Inset: $V = 0$ differential conductance. Solid line: Prediction for our finite length wire. Dotted line: $T^{-1/2}$ dependence expected for an infinite wire.

an electron is added at the surface of the wire, the potential establishes itself in the thickness of the wire on a time scale given by the reciprocal of the plasma frequency. After this transient, too short to play a role given our energy scale, the electrical potential is homogeneous within the wire thickness, and diffuses in the two other dimensions, with a diffusion constant $D^* = (R_\square C_s)^{-1} = (R_l C_l)^{-1} \sim 10\ \mathrm{m}^2/\mathrm{s}$, where R_l and C_l are the resistance and capacitance per unit length of the wire, and C_s is the capacitance per unit area [17]. The electron itself diffuses much more slowly, with a diffusion constant $D = (\nu e^2 w a R_l)^{-1} \sim 27 \times 10^{-4}\ \mathrm{m}^2/\mathrm{s}$, where w and a are the width and thickness of the wire, and ν the density of states at the Fermi energy. The complete calculation of $Z_{\mathrm{eff}}(\omega)$ in the case of an infinitely long junction shows that the spreading of the potential in the thickness and in the width of the wire can be taken as instantaneous at energies $eV \ll \hbar D^*/w^2 \sim 100$ meV. Note that the dimensionality criterion is not determined by the comparison of eV with the Thouless energy $\hbar D/w^2$, as is usually assumed [7].

In the one-dimensional regime relevant to our experiment, we first treat the case of a junction with infinite length. The impedance is then that of two RC semi-infinite transmission lines in parallel: $z(x_0, x, \omega) = \frac{1}{2} Z_c(\omega) e^{-\gamma(\omega)|x-x_0|}$, where $Z_c(\omega) = \sqrt{R_l/iC_l\omega}$ is the characteristic impedance of one transmission line, $\gamma(\omega) = \sqrt{i\omega/D^*}$ and x the coordinate along the wire. The expression for $p(0, x, \omega) = \frac{1}{2}\sqrt{1/iD\omega}\, e^{-\sqrt{i\omega/D}|x-x_0|}$ has the same form, since both the electric potential and the electron probability cloud obey a diffusion equation in our system, but D now enters in place of D^*. We then obtain for the real part of the effective impedance: $\mathrm{Re}Z_{\mathrm{eff}}(\omega) = \frac{1}{2}\,\mathrm{Re}Z_c(\omega)/(1 + \sqrt{D/D^*})$. The correction due to electron diffusion, of order $\sqrt{D/D^*} \approx 0.02$ [18], is thus negligible and in the following we take $Z_{\mathrm{eff}}(r_0, \omega) = z(x_0, x_0, \omega)$. Our junction can now be modeled as a ladder array of infinitesimal junctions and resistances, each junction seeing an environmental impedance due to the resistances and the other junction capacitances (see Fig. 1b). Such a simplification does not occur for two-dimensional junctions, where the interplay of electron and potential diffusion cannot be neglected and where $Z_{\mathrm{eff}}(\omega)$ includes the nontrivial factor $\ln(D^*/D)$ [7,8,19].

We thus find, in the case of the infinite wire, at very low or very large voltages, the following analytical expressions:

$$\delta G/G_t \simeq -1.56 \frac{R_l}{R_K}\sqrt{\frac{\hbar D^*}{k_B T}} \quad \text{for } \frac{eV}{k_B T} \to 0\,,$$

while

$$\delta G/G_t = -\sqrt{2}\,\frac{R_l}{R_k}\sqrt{\frac{\hbar D^*}{eV}} \quad \text{for } \frac{eV}{k_B T} \to \infty\,.$$

The crossover between these two expressions occurs for $eV \sim 10 k_B T$.

In the case of a wire with finite length with a terminal impedance, the full voltage and temperature dependence can be computed numerically using $z(x_0, x_0, \omega)^{-1} = Z_{x_0}(\omega)^{-1} + Z_{L-x_0}(\omega)^{-1}$, where $Z_x(\omega)/Z_c(\omega) = (\rho + \theta)/(1 + \rho\theta)$ with $\rho = Z_0(\omega)/Z_c(\omega)$ and $\theta = \tanh[\gamma(\omega)x]$. These last expressions have been used to fit the complete data set with three parameters: the capacitance per unit length C_l, the tunnel conductance G_t, and the terminal impedance Z_0. The best fit, obtained with $G_t = 26.392\ \mu\mathrm{S}$, $C_l = 7.4\ \mathrm{fF}/\mu\mathrm{m}$, and $Z_0 = 50\ \Omega$ is shown with solid lines on Fig. 2 and the right half of Fig. 3 for three temperatures. In Fig. 2, we also show the predictions for the infinite wire, both at $T = 0$ and at the experiment temperature. At large voltages, the $1/\sqrt{V}$ dependence [1] is well observed, down to voltages corresponding to the predicted crossover. Note that at low voltages, the effect of the finite length of the wire is a rather small correction, except at the lowest temperatures and voltages. In the inset of Fig. 3 we show the predictions for the $V = 0$ conductance as a function of temperature. The infinite wire prediction (dotted curve) gives only a qualitative account of the data. We find good agreement between the full theory and experiment at temperatures above 50 mK, whereas for the lowest temperature, the dip is not as pronounced as predicted. This discrepancy can be attributed to a slight heating of the electrons: a good fit of the 26 mK data is obtained with a 37 mK theoretical curve. This problem should not cast doubt on the rest of the data set: we believe heating cannot affect $G(V, T)$ above 50 mK as currents in the junction are not sufficient to raise significantly the temperature of the electrodes, given the known electron-phonon coupling. The influence of external parasitic noise can likewise be eliminated from other measurements done in the same setup. The value of C_l is about 3× smaller than what is usually measured for aluminum junctions. However, the very low conductance per unit area of the tunnel junction, indicating a very thick insulating layer obtained from an intensive oxidation, is a plausible explanation for the difference. Finally, the value of the fit parameter Z_0 agrees with estimates and values found in previous experiments [6].

It is worth stressing that the $1/\sqrt{V}$ and $1/\sqrt{\mathrm{T}}$ dependences observed here arise from the diffusion of the electric potential along the junction and *not* from that of diffusive quasiparticles interacting only via a short-range interaction.

In conclusion, we have found that the tunneling conductance of an extended junction shows the dip near zero voltage expected from the macroscopic electrodynamic contribution of electron-electron interaction. Our results are easily explained quantitatively using the concept of distributed impedance borrowed from Coulomb blockade effects and by considering that the junction contributes to its own electrodynamic environment. An important prediction of this approach is that the electrodynamic dip observed in our experiment is washed out if electrons diffuse coherently fast enough away from the region where the resistance is concentrated. This prediction could not be tested in the one-dimensional experiment we have performed, but could in principle be tested, for instance, by low temperature STM tunnel spectroscopy with a tip on a two-dimensional sample.

We gratefully acknowledge discussions with I. Aleiner, B. Altshuler, M. Feigelmann, L. Glazman, H. Grabert, G. Ingold, L. Levitov, Yu. Nazarov, and E. Sukhorukov. This work has been partly supported by the Bureau National de la Métrologie. N. O. B. acknowledges NSF support under Grant No. DMR-9801841. Travel between East Lansing and Saclay was supported by NATO, under Collaborative Research Grant No. 970273.

*Permanent address: Michigan State University, East Lansing, MI 48824.

[1] B.L. Altshuler and A.G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)]; B.L. Altshuler, and A.G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A.L. Efros and M. Pollak (Elsevier Science Publishers B.V., Amsterdam, 1985); P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[2] R.C. Dynes and J.P. Garno, Phys. Rev. Lett. **46**, 137 (1981); Y. Imry and Z. Ovadyahu, Phys. Rev. Lett. **49**, 841 (1982); Alice E. White, R.C. Dynes, and J.P. Garno, Phys. Rev. B **31**, 1174 (1985); M.E. Gershenzon V.N. Gubankov, and M.I. Falei, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 435 (1985) [JETP Lett. **41**, 534 (1985)]; Alice E. White, R.C. Dynes, and J.P. Garno, Phys. Rev. Lett. **56**, 532 (1986); Shih-Ying Hsu and J.M. Valles, Jr., Phys. Rev. B **49**, 16 600 (1994); J.G. Massey and Mark Lee, Phys. Rev. Lett. **77**, 3399 (1996).

[3] M.H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, and C. Urbina, Phys. Rev. Lett. **64**, 1824 (1990); S.M. Girvin, L.I. Glazman, M. Jonson, D.R. Penn, and M.D. Stiles, Phys. Rev. Lett. **64**, 3183 (1990).

[4] G.-L. Ingold and Yu.V. Nazarov, in *Single Charge Tunneling,* edited by H. Grabert and M.H. Devoret (Plenum Press, New York, 1992).

[5] P. Delsing, K.K. Likharev, L.S. Kuzmin, and T. Claeson, Phys. Rev. Lett. **63**, 1180 (1989); A.N. Cleland, J.M. Schmidt, and J. Clarke, Phys. Rev. Lett. **64**, 1565 (1990); J.P. Kauppinen and J.P. Pekola, Phys. Rev. Lett. **77**, 3889 (1996); P. Wahlgren, P. Delsing, and T. Claeson, Phys. Rev. B **57**, 2375 (1998); P. Joyez, D. Esteve, and M.H. Devoret, Phys. Rev. Lett. **80**, 1956 (1998).

[6] T. Holst, D. Esteve, C. Urbina, and M.H. Devoret, Phys. Rev. Lett. **73**, 3455 (1994).

[7] B.L. Altshuler, A.G. Aronov, and A.Yu. Zyuzin, Zh. Eksp. Teor. Fiz. **86**, 709 (1984) [Sov. Phys. JETP **59**, 415 (1984)]. The statement in [8] that the results from this paper and the Coulomb blockade theory are different is not correct.

[8] Appendix in [4], p, 91.

[9] A. Kamenev and A. Andreev, Phys. Rev. B **60**, 2218 (1999).

[10] A.M. Rudin, I.L. Aleiner, and L. Glazman, Phys. Rev. B **55**, 9322 (1997).

[11] Yu.V. Nazarov, Zh. Eksp. Teor. Fiz. **95**, 975 (1989) [Sov. Phys. JETP **68**, 561 (1990)].

[12] E.V. Sukhorukov and A.V. Khaetskii, Phys. Rev. B **56**, 1456 (1997); G.M. Minkov, A.V. Germanenko, S.A. Negachev, O.E. Rut, and E.V. Sukhorukov, Phys. Rev. B **59**, 13 139 (1999).

[13] We have here corrected a sign in [8] [Yu.V. Nazarov and B.L. Altshuler (private communication)].

[14] G.J. Dolan and J.H. Dunsmuir, Physica (Amsterdam) **152B**, 7 (1988). Such large junctions were obtained with a very thick bilayer of MAA (5 μm) and PMMA (0.5 μm).

[15] D. Vion, P.F. Orfila, P. Joyez, D. Esteve, and M.H. Devoret, J. Appl. Phys. **77**, 2519 (1995).

[16] W.F. Brinkmann, R.C. Dynes, and J.M. Rowell, J. Appl. Phys. **41**, 1915 (1970). A fit to this theory gives for the thickness of the tunnel barrier $\delta = 1.8$ nm, and for the heights of the barrier, found to be asymmetric, $\varphi_1 = 2.4$ eV and $\varphi_2 = 3.5$ eV.

[17] We neglect the inductance of the wire, estimated to be 0.2 pH/μm.

[18] The value of $D^*/D = e^2 v \delta a/\varepsilon$ is at least 1000 in thin-film metallic junctions: ε and δ are the dielectric constant and thickness of the oxide layer.

[19] L.S. Levitov and A.V. Shytov, JETP Lett. **66**, 215 (1997).