

## Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction

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The escape rate of an underdamped ( $Q \approx 30$ ), current-biased Josephson junction from the zero-voltage state has been measured. The relevant parameters of the junction were determined *in situ* in the thermal regime from the dependence of the escape rate on bias current and from resonant activation in the presence of microwaves. At low temperatures, the escape rate became independent of temperature with a value that, with no adjustable parameters, was in excellent agreement with the zero-temperature prediction for macroscopic quantum tunneling.

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The observation of macroscopic quantum tunneling is regarded as a test of whether quantum mechanics is valid for macroscopic variables, a fundamental question<sup>1</sup> that has only recently been addressed experimentally. The necessary conditions for the observation of macroscopic quantum tunneling can be realized in the current-biased Josephson tunnel junction, where the phase difference between the two superconductors is the macroscopic variable, and the tunneling occurs from the zero-voltage state to the nonzero-voltage state. Previous experiments on a current-biased Josephson junction<sup>2-4</sup> or on a superconducting ring interrupted by a Josephson junction<sup>5-7</sup> have yielded results that have been interpreted as being consistent with the theoretical predictions for macroscopic quantum tunneling. In this Letter, we present results of experiments on a current-biased junction that differ from earlier measurements primarily in that we determine *in situ* all of the relevant parameters using *classical* phenomena. In particular, we measure the impedance shunting the junction at the relevant microwave frequencies. We are thus able to compare the experimental results quantitatively with theoretical predictions with no adjustable parameters.

The current-biased Josephson junction can be represented as a particle moving in a one-dimensional tilted cosine potential.<sup>8</sup> The zero-voltage state of the junction corresponds to the confinement of the particle to one well of this potential. After the particle escapes from this metastable state, it runs freely down the tilted cosine potential, and a voltage appears across the junction. For a constant bias current  $I$  slightly less than the critical current  $I_0$ , the well from which the particle escapes is given by a cubic potential with barrier height<sup>9</sup>  $\Delta U = (2\sqrt{2}I_0\Phi_0/3\pi)(1 - I/I_0)^{3/2}$ , where  $\Phi_0 = h/2e$  is the flux quantum. We have designed the experiment so that the total admittance across the junction, including contributions from the current and voltage leads, can be represented to a good approximation by a capacitance  $C$  and a resistance  $R$  in parallel.

In the zero-voltage state, the plasma frequency  $\omega_p/2\pi$  of small oscillations of the particle at the bottom of the well is  $\omega_p = (2\pi I_0/C\Phi_0)^{1/2}[(1 - (I/I_0)^2)]^{1/4}$ , while the damping factor is  $Q = \omega_p RC$ .

In the thermal regime ( $k_B T \gg \hbar\omega_p$ ), the escape of the particle from the well occurs via thermal activation at a rate<sup>10</sup>

$$\Gamma_t = a_t(\omega_p/2\pi)\exp(-\Delta U/k_B T), \quad (1)$$

where  $a_t = 4/[(1 + Qk_B T/1.8\Delta U)^{1/2} + 1]^2$  is of the order of unity in our experiment,  $k_B$  is Boltzmann's constant, and  $T$  is the temperature. In the quantum regime ( $k_B T \ll \hbar\omega_p$ ), to lowest order in  $1/Q$  the escape is predicted to occur via macroscopic quantum tunneling at a rate<sup>11</sup>

$$\Gamma_q = \frac{a_q \omega_p}{2\pi} \exp\left[-\frac{7.2\Delta U}{\hbar\omega_p} \left(1 + \frac{0.87}{Q}\right)\right] \quad (2)$$

at  $T = 0$ , where  $a_q \approx [120\pi(7.2\Delta U/\hbar\omega_p)]^{1/2}$ .

To express the experimental measurements of the escape rate in a way that is as independent as possible of the parameters of the junction, we introduce the "escape temperature"  $T_{\text{esc}}$  defined through the relation

$$\Gamma = (\omega_p/2\pi)\exp(-\Delta U/k_B T_{\text{esc}}). \quad (3)$$

In the thermal regime, the theoretical prediction is

$$T_{\text{esc}} = T/(1 - p_t), \quad (4)$$

where the magnitude of  $p_t = (k_B T/\Delta U)\ln a_t$  is small compared with unity. In the quantum regime at  $T = 0$ , the prediction is

$$T_{\text{esc}} = \frac{\hbar\omega_p/k_B}{7.2(1 + 0.87/Q)(1 - p_q)}, \quad (5)$$

where  $p_q \approx (\hbar\omega_p/7.2\Delta U)\ln a_q$ . The crossover temperature at which the escape rate changes from thermal (temperature dependent) to quantum (tempera-

ture independent) is predicted<sup>12</sup> to be  $\hbar\omega_p/2\pi k_B$  in the limit  $Q \gg 1$ . We will determine  $T_{\text{esc}}$  from the measured values of  $\Gamma$ ,  $\omega_p$ ,  $I_0$ , and  $I$ , and compare these experimental values with that predicted by Eq. (4) or (5).

The experimental configuration has been described previously.<sup>13</sup> The  $10 \times 10\text{-}\mu\text{m}^2$  Nb-NbO<sub>x</sub>-PbIn tunnel junction was mounted at one end of an attenuating coaxial line. The mount and the last of a series of low-pass filters for the bias circuitry were thermally anchored to the mixing chamber of a dilution refrigerator. The critical current of the junction could be reduced by a magnetic field. A separate, heavily filtered coaxial line, capacitively coupled to the bias leads of the junction, could be used to inject a microwave bias current, thereby enabling us to measure  $\omega_p(I)$  and  $Q(I)$  *in situ* in the classical limit using resonant activation.<sup>13</sup> We determined the escape rates by ramping the bias current and measuring the value of current at which the junction switched to the nonzero-voltage state.<sup>9</sup> By turning off the current within 30  $\mu\text{s}$  after the transition, we were able to make heating effects negligible. We collected typically  $10^5$  switching events for each measurement of  $\Gamma$ .

The critical current was determined in the thermal regime by the exponential dependence of the escape rate on the bias current. As is evident from the expressions for  $\Delta U$  and  $\Gamma$ , a plot of the experimentally determined quantity  $[\ln(\omega_p(I)/2\pi\Gamma)]^{2/3}$  vs  $I$  should, with neglect of departures of  $a_t$  from unity, be a straight line with slope scaling as  $T_{\text{esc}}^{2/3}$  that intersects the current axis at  $I_0$ . Figure 1 shows three examples of such plots out of the seven obtained in the thermal regime over the temperature range from 102 to 800 mK. In addition, we show two sets of data obtained at 46 and 19 mK where quantum corrections are expected to be important. We note that the slope

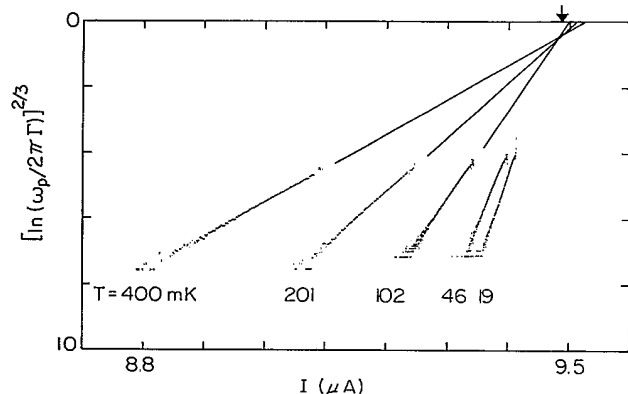


FIG. 1.  $[\ln(\omega_p/2\pi\Gamma)]^{2/3}$  vs  $I$  for five values of temperature. Lines that intersect the current axis have been drawn through the data in the thermal regime, at the three highest temperatures. The arrow indicates the value of  $I_0$  obtained after corrections for the prefactor were made.

changes very little as the temperature is lowered from 46 to 19 mK, indicating that  $T_{\text{esc}}$  is nearly the same at these two temperatures. In fact, two sets of data obtained at 30 and 24 mK (not shown in Fig. 1) were indistinguishable from the data at 19 mK. As expected, the lines drawn through the data at the three higher temperatures intersect the current axis at very nearly the same point. The values of  $I_0$  obtained from the seven sets of data in the thermal regime ranged from 9.498 to 9.535  $\mu\text{A}$ . We then corrected these values of  $I_0$  for the departure of  $a_t$  from unity, using the value  $Q = 30 \pm 15$  obtained from resonant activation.<sup>13,14</sup> These corrections were small, varying from  $-12 \pm 4$  nA at 100 mK to  $-47 \pm 12$  nA at 800 mK. After these corrections were made, the critical current was independent of temperature to within the experimental uncertainties, with the value  $9.489 \pm 0.007 \mu\text{A}$ . Quantum corrections<sup>15</sup> to  $I_0$  were negligible.

We used the measured values of  $\omega_p(I)$  and  $Q(I)$  at numerous values of  $I$  and three values of  $I_0$  to determine  $C$  and  $R$  as functions of  $\omega_p/2\pi$  over the frequency range from 2 to 8 GHz; a typical value of  $\omega_p/2\pi$  for the high-critical-current junction in the quantum limit was 4 GHz. We found that the measured value of both  $C$  and  $R$  varied somewhat over this frequency range, presumably because of standing-wave resonances in the line connected to the junction. However, the variations in  $C$  were sufficiently small that we feel justified in using an average value with an error bar that includes most of the variation,  $C = 6.35 \pm 0.4$  pF. The frequency dependence of the value of  $R$  was more pronounced, and we have taken the value  $R = 190 \pm 100 \Omega$ , where the quoted error again includes most of the variation. Since the damping in these experiments is relatively weak, the large uncertainty in  $R$  does not lead to a significant error in the predicted value of  $T_{\text{esc}}$  at  $T=0$ . As in our previous experiments,<sup>13</sup> the value of  $R$  was dominated by the conductance of the coaxial line to which the junction was attached, while  $C$  was dominated by the self-capacitance of the junction.

Using the measured values of  $I_0$ ,  $C$ , and  $R$ , we can compute<sup>16</sup>  $T_{\text{esc}}$  from  $\Gamma$  as a function of  $I$  and  $T$ . In Fig. 2, we plot  $T_{\text{esc}}$  vs  $T$  for a junction with  $I_0 = 9.489 \pm 0.007 \mu\text{A}$ . Since, as we shall see,  $T_{\text{esc}}$  is weakly dependent on the bias current, we have plotted these data at a bias current chosen so that  $\ln(\omega_p/2\pi\Gamma) = 11$ . The predicted crossover temperature of 30 mK is indicated by a solid arrow in Fig. 2. At temperatures above about 100 mK, the measured value of  $T_{\text{esc}}$  is very close to the temperature  $T$  as we expect in the thermal regime. At temperatures below about 25 mK, on the other hand,  $T_{\text{esc}}$  becomes independent of temperature, with a value of  $37.4 \pm 4$  mK. The Caldeira-Leggett prediction at  $T=0$  is  $T_{\text{esc}} = 36.0 \pm 1.4$  mK, which is in very good agreement with the tem-

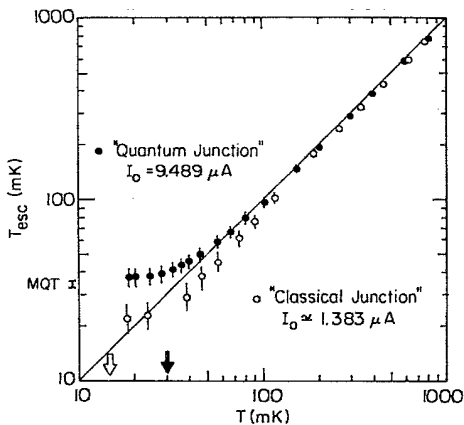


FIG. 2.  $T_{\text{esc}}$  vs  $T$  for two values of critical current for  $\ln(\omega_p/2\pi\Gamma) = 11$ . The solid and open arrows indicate the predicted crossover temperatures for the higher and lower critical currents, respectively. The prediction of Eq. (5) for the higher critical current is indicated at the left.

perature-independent value observed in our experiment. The contribution of the damping to the predicted value of  $T_{\text{esc}}$  is  $-1.5$  mK, which is less than the combined uncertainty of the theoretical prediction and experiment. Thus we cannot presently make any statement about the effect of dissipation on quantum tunneling. We note that the error in the measured values of  $T_{\text{esc}}$  in the quantum limit is dominated by the uncertainty in  $\Delta U$ , which arises, in turn, from the uncertainty in  $I_0$ . On the other hand, the error in the predicted value of  $T_{\text{esc}}$  arises predominantly from uncertainties in  $\omega_p$  and  $Q$ .

Although the low-temperature values of  $T_{\text{esc}}$  plotted in Fig. 2 are in good agreement with the  $T = 0$  prediction, nevertheless one should demonstrate that the flattening of  $T_{\text{esc}}$  is not due to an unknown, spurious noise source. To establish that the effective temperature of the dissipative element was close to  $T$  down to the lowest temperatures of the experiment, we applied a magnetic field to the junction to reduce the critical current. After we had corrected the data for the temperature dependence of  $a_t$ , we found that this reduced critical current still varied very slightly with temperature, from  $1.376 \pm 0.005 \mu\text{A}$  at 800 mK to  $1.388 \pm 0.002 \mu\text{A}$  at 20 mK. The temperature dependence of  $I_0$  may have arisen because of the sensitivity of  $I_0$  to magnetic field and the fact that the applied field possibly changed with temperature. In Fig. 2, we have also plotted  $T_{\text{esc}}$  for the junction with the lower critical current for  $\ln(\omega_p/2\pi\Gamma) = 11$ . At each temperature, we calculated  $T_{\text{esc}}$  using the value of  $I_0$  measured at that temperature. The predicted crossover temperature, 14 mK, is indicated with an open arrow. We observe that  $T_{\text{esc}}$  is equal to  $T$  to within the experimental error, although there is a suggestion that  $T_{\text{esc}}$  is begin-

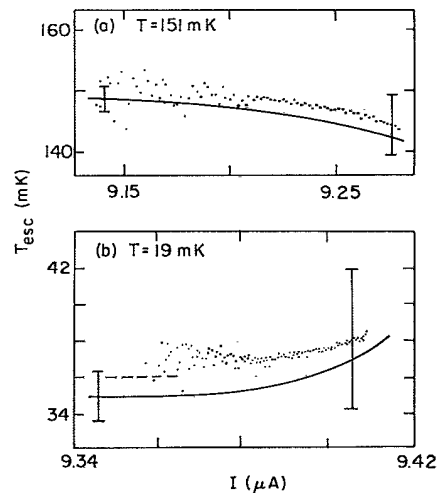


FIG. 3.  $T_{\text{esc}}$  vs  $I$  for a junction with  $I_0 = 9.489 \pm 0.007 \mu\text{A}$  (a) in the classical regime and (b) in the quantum regime. Points are the experimental data and solid lines are the theoretical prediction. The dashed line in (b) is the prediction for zero damping. The error bar on the left and the right of each figure represents the possible shift in the theoretical and experimental curves, respectively, due to uncertainties in the experimental parameters. The solid line represents  $T_{\text{esc}} = T$ .

ning to flatten off at the lowest temperature, where quantum effects are likely to become significant. Thus we conclude that the flattening of  $T_{\text{esc}}$  for the junction with the higher critical current did not arise from spurious noise sources.

An important difference between the thermal and quantum regimes may be observed through the weak dependence of  $T_{\text{esc}}$  on the bias current, which arises from the different forms of  $a_t$  and  $a_q$  and from the current dependence of  $\omega_p$ . This behavior is illustrated in Fig. 3 for  $I_0 = 9.489 \mu\text{A}$ . In Fig. 3(a) we plot  $T_{\text{esc}}$  vs  $I$  in the thermal regime ( $T = 151$  mK), together with the prediction of Eq. (4). The decrease of  $T_{\text{esc}}$  with increasing bias current arises because  $a_t < 1$ . Within the uncertainties, the data are in good agreement with theory. Figure 3(b) shows  $T_{\text{esc}}$  vs  $I$  in the quantum regime ( $T = 19$  mK), together with the prediction of Eq. (5). In this limit,  $T_{\text{esc}}$  increases with increasing bias current through the current dependence of  $\Delta U$  because  $a_q \gg 1$ ; the current dependence of  $\omega_p$  is relatively unimportant. Again, within the experimental uncertainties, the data are in good agreement with theory. The very different current dependence of  $T_{\text{esc}}$  at low and high temperatures lends further support to the claim that the escape mechanisms are different in the two temperature regimes.

In summary, we have measured the escape rate of a current-biased, underdamped ( $Q \approx 30$ ) Josephson tunnel junction from the zero-voltage state for two values of critical current, the lower value being

achieved by means of an external magnetic field. At the lower critical current,  $T_{\text{esc}}$  followed the classical prediction to within experimental error, indicating that the effects of extraneous noise were negligible. For the higher value of critical current, the value of  $T_{\text{esc}}$  was equal to  $T$  at high temperatures, but began to flatten off at temperatures below 50 mK and became independent of temperature below 25 mK. Within the experimental uncertainties, the low-temperature value of  $T_{\text{esc}}$  was in excellent agreement with the theoretical prediction for  $T=0$ , with all the relevant parameters measured *in situ* in the classical limit.

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<sup>16</sup>For  $T > 400$  mK, it was necessary to use the full rather than the approximate expression for  $\Delta U$ .