Observation of Parity-Induced Suppression of Josephson Tunneling in the Superconducting Single Electron Transistor

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We have measured the supercurrent branch of a superconducting single electron transistor as a function of gate charge, temperature, and magnetic field. At low temperature and magnetic field, the switching current goes from a minimum to a maximum when the gate charge is varied from 0 to e, as expected for an island in the ground state with an even electron number. When the odd electron number ground state becomes populated by an increase of temperature or field, the Josephson tunneling is strongly suppressed, in agreement with theoretical predictions.

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The consequences of the duality of phase and number-of-particle variables are particularly well illustrated by the competition between Josephson tunneling and single electron charging phenomena in ultrasmall superconducting junction systems [1,2]. One of the simplest devices consists of two Josephson junctions in series [3–6]: The number of Cooper pairs on the middle "island" tends to be fixed by the charging energy $E_C=e^2/2C_\Sigma$ of the island while the associated phase tends to be fixed by the Josephson coupling energy E_J of the two junctions which we suppose identical for simplicity. Here C_Σ refers to the total capacitance of the island. This model system has been investigated theoretically in detail [1,7–9]. For large area junctions ($E_J \gg E_C$) the charging effects are overcome by Josephson tunneling and the maximum supercurrent that can flow through the two junction system is just $I_0=2eE_J/\hbar$, the maximum supercurrent of each junction. However, for small area junctions ($E_J \ll E_C$), the maximum supercurrent should strongly depend on the polarization charge Q_g applied to the island by means of a gate electrode, hence the name of "superconducting single electron transistor" given to such device. When $Q_g = e \bmod 2e$, i.e., when states differing by one Cooper pair in the island are degenerate, the maximum current should attain $I_0/2$ while for $Q_g=0 \bmod 2e$ it should fall to a value of order I_0E_J/E_C [1] (here and in the following, we assume for convenience that the neutral island has an even number of electrons). Recently Matveev *et al.* [9] have shown theoretically that this simple electrostatic modulation of Josephson tunneling will be observed only if the parity of the number n of excess electrons on the island can be kept even for all Q_g. This requires that the odd-even free energy difference D [5,10] of the island is greater than E_C. When $D<E_C$, the island is unstable, in the vicinity of $Q_g=e$, with respect to the entrance of a quasiparticle. This quasiparticle prevents the formation of the coherent superposition of charge states at $Q_g=e$, and therefore "poisons" Josephson tunneling. A complex Q_g dependence of the supercurrent should then be observed. In this Letter we present an experiment on the superconducting single electron transistor in which, for the first time, we observe the characteristic features resulting from poisoning of Josephson tunneling.

The sample was prepared using standard e-beam lithography and shadow mask evaporation techniques [11]. The main difference with previous experiments is the use of the three-angle evaporation technique of Haviland *et al.* [12] in order to fabricate in a single pump down the alumina-covered Al island electrode, the two Al drain and source electrodes, and the Cu (3 wt.% Al) buffer electrodes (see device layout in the inset of Fig. 1). We believe that these last electrodes allow the quasiparticle population in the transistor to reach the thermal equilibrium value and prevent uncontrolled poisoning of Josephson tunneling by out-of-equilibrium quasiparticles from the rest of the circuit. The contact between the Cu and Al electrodes is sufficiently good to have a negligible influence on the behavior of the transistor at low voltages. The electrical wiring between the sample and the measuring apparatus at room temperature was made through a series of cryogenic filters as in previous experiments [10]. From the measurement of the device with the Al electrodes brought in the normal state by a magnetic field, we obtained the relation between the gate charge Q_g and gate voltage U, and we could estimate $E_C/k_B=1.0$ K. The normal resistance of the two junctions in series was $R_N=49.2$ kΩ. The value $\Delta=180$ μeV of the gap of the superconducting aluminum was extracted from the large scale I-V characteristic of the sample in zero magnetic field. Using the Ambegaokar-Baratoff relation [13], we deduced from R_N and Δ the Josephson energy $E_J/k_B=275$ mK and critical current $I_0=2eE_J/\hbar=11.4$ nA of each junction, supposing they are identical. In Fig. 1 we show the subgap current-voltage (I-V) characteristic of the device at $T=20$ mK and for $Q_g \simeq e$. A supercurrent branch is clearly seen with nearly zero voltage like in the recent experiment by Eiles and Martinis [6]. Its residual slope was measured to be less than 100 Ω, our resistance resolution given the wiring of the sample to the external apparatus. This branch defines a switching current I_s at which the device switches to a voltage set by the resistance of the current bias source, which was 12.1 MΩ for the data we present in the remainder of this paper.

In Fig. 2(a) we show the variations of I_s as a function

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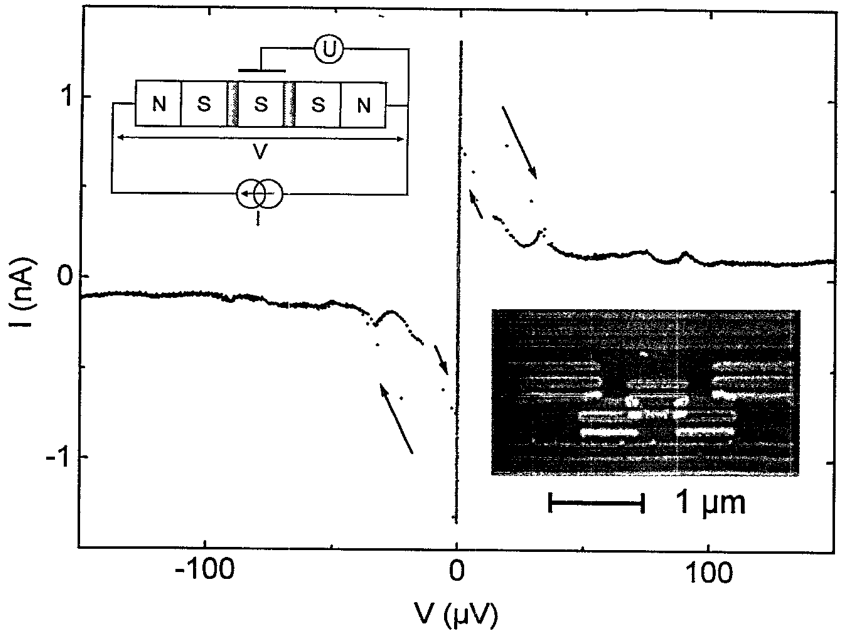

FIG. 1. Current-voltage characteristic of superconducting single electron transistor whose layout is shown in upper left inset. The letters N and S refer to normal (Cu) and superconducting (Al) electrodes. The tunnel barriers are indicated by grey rectangles. The gate voltage U induces on the middle island a gate charge Q_g whose value is e for the data shown. The temperature was 20 mK. The maximum current defines the switching current I_s. Lower right inset is an electron micrograph of the device. The current flows through the middle strip only. The top electrode is the gate.

of the gate charge Q_g for several values of the magnetic field and at $T=65$ mK. At lower temperatures the data did not change except for Q_g/e in the vicinity of $\pm 0.75 \times \text{mod} 2$ where we observed what we interpret as a low voltage self-induced Shapiro step [14] and which slightly biased the measurement of the switching current. At low magnetic fields, the switching current varied monotonically when the gate charge was varied from 0 to e. As the field increased, the peak at $Q_g=e$ became a dip, a behavior corresponding to the poisoning of Josephson tunneling by a single quasiparticle. This dip widened as the field was increased further, in agreement with Ref. [9].

In order to compare our experimental results to theory, we now make a minimal extension of Ref. [9] to take into account finite temperature and environmental impedance. The states of the transistor are conveniently characterized by two quantum numbers, the number $n=N-N'$ of excess electrons on the island and by the charge flow index $k=(N+N')/2$, where N and N' denote the number of electrons having crossed the junctions [see Fig. 3(a)]. The Josephson Hamiltonian couples states with different k but with the same parity of n and we can thus separate the manifold of states into odd-n and even-n manifolds. In the following, the superscript p will designate a given parity, even or odd. Inside a manifold of parity p, we now perform a change of representation, in which the

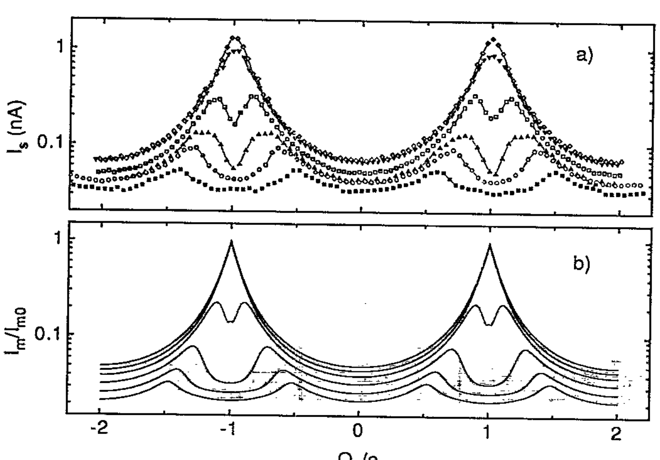

FIG. 2. (a) Switching current as a function of gate charge, for several values of the magnetic field H, at $T=65$ mK. Top to bottom: $H=0$, 0.07, 0.11, 0.14, 0.16, 0.17 T. The dip at odd integer values of Q_g/e corresponds to the poisoning of Josephson tunneling by the entrance of one quasiparticle in the island. (b) Theoretical runaway current as a function of gate charge, for the same field values as in (a).

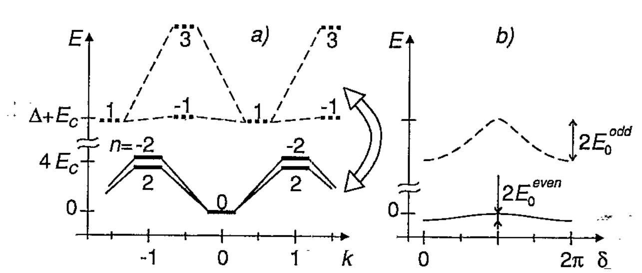

FIG. 3. (a) Energy levels of the transistor at $Q_g=0.02e$. The numbers n labeling the levels refer to the number of electrons in the middle island. The number k is the charge transfer index. The lines joining the levels represent the Josephson coupling. Only levels with the same parity of n are coupled. The even-n manifold (levels in solid line) and the odd-n manifold (levels in dashed line) are weakly coupled by the cotunneling of one electron from a normal lead to the middle island (double arrow). (b) Lowest energy bands corresponding to the even and odd manifolds. The variable δ is canonically conjugate to k.

new states are indexed by n and δ, the total phase difference of the transistor, which is the variable canonically conjugate to k. If we now restrict the span of n to the three lowest electrostatic energy states, we can exactly diagonalize the sum of the Josephson and electrostatic Hamiltonians. In contrast with the treatment of Ref. [9], this procedure takes into account the degeneracy of the first excited charge states that occurs at $Q_g=0$ ($=e$) when p is even (odd). We obtain a ground state energy band $E_0^g f_0^g(\delta)$, where the function f_0^g is such that $\mathrm{Max}\{f_0^g\}-\mathrm{Min}\{f_0^g\}=2$, for arbitrary values of the parameters E_J, E_C, and Q_g [see Fig. 3(b)]. In this calculation we assume a gate voltage invariant Josephson coupling E_J for each of the junctions (this is valid since the energy gap Δ of the superconductor is such that $E_C \ll 2\Delta$). The 2π-periodic $E_0^g f_0^g(\delta)$ function is equivalent for the transistor to the energy-phase relation $-E_J \times\cos(\delta)$ for a single Josephson junction; in particular it goes from a minimum to a maximum when δ goes from 0 to π. The transistor can thus be seen as an effective junction with a gate charge-dependent effective Josephson coupling energy E_0^g.

The relation between the I-V characteristic observed experimentally and the energy-phase relation depends on both the temperature T and the admittance $Y(\omega)$ which, in the lumped element model of the electromagnetic environment of the junction, is in parallel with the bias current source I. This admittance will govern the dynamics of δ which is analogous to that of a particle in the tilted potential $E_0^g f_0^g(\delta)-(\Phi_0/2\pi)\delta I$, where $\Phi_0=h/2e$. In the case of interest here, where the response time of the admittance is short compared to the characteristic time of the evolution of δ, we can write the differential equation obeyed by δ as

$$\frac{\Phi_0}{2\pi}\left[Y(0)\dot{\delta}-jY'(0)\ddot{\delta}-\frac{1}{2}Y''(0)\dddot{\delta}+\cdots\right]+\frac{2\pi E_0^g}{\Phi_0}\frac{df_0^g}{d\delta}=I\,.$$

This equation generalizes the equation of motion of the resistively and capacitively shunted junction (RCSJ) model [15] to an effective Josephson element shunted by a general admittance.

For $I\le I_{c0}=(2\pi E_0^g/\Phi_0)\mathrm{Max}\{df_0^g/d\delta\}$, this equation admits a zero-voltage solution ($\dot{\delta}=0$) corresponding to the particle sitting in a minimum of the tilted potential. This solution is unstable against thermal fluctuations and therefore the particle will diffuse from well to well in the potential, giving rise to a departure of the supercurrent branch from the zero-voltage axis. However, for $I_m\le I<I_{c0}$ this diffusive motion is itself unstable against the runaway down the potential [16], where I_m is the current for which, on the average, the energy gain due to the tilt of the potential becomes greater than energy loss due to friction. In the weak friction limit appropriate to our experiment, the runaway current I_m is given by

$$I_m=\Phi_0\left[\alpha Y(0)\left(\frac{E_0^g}{\Phi_0^2|Y'(0)|}\right)^{1/2}+\beta Y''(0)\left(\frac{E_0^g}{\Phi_0^2|Y'(0)|}\right)^{3/2}+\cdots\right],$$

where $\alpha,\beta,\ldots$ are dimensionless coefficients which are weakly dependent on f_0^g. The first term in the expansion corresponds to the well known $4I_0/\pi RC\omega_p$ result of the RCSJ model [17]. Here, since we have an unshunted junction, this term vanishes and the Q_g dependence of I_m is dominated by the second term. In view of the importance of thermal fluctuations in our experiment ($E_0^g \le E_J/2$), we will compare the Q_g dependence of the measured switching current with the theoretical Q_g dependence of I_m rather than of the critical current I_{c0} considered by Matveev *et al.* [9].

We now make a crucial assumption. We assume that the inverse of the transition rate between the odd-n and even-n states is much smaller than the characteristic time of the runaway process. This assumption of rapid odd-even transition is justified since the normal electrodes, which provide the quasiparticle involved in the transition, are very close to the island [18]. In the calculation of the switching current, we thus replace E_0^g by the Boltzmann average $E_0^{\mathrm{av}}=E_0^{\mathrm{odd}}p_{\mathrm{odd}}+E_0^{\mathrm{even}}p_{\mathrm{even}}$ where p_{odd} and p_{even} are the probabilities of being in an odd- or even-n state, respectively, and which verify

$$p_{\mathrm{odd/even}}\propto\sum_{n\ \mathrm{odd/even}}\exp\{-[E_C(Q_g/e-n)^2+(n\,\mathrm{mod}2)D(T,H)]\}/k_BT\,.$$

Here $D(T,H)$ is calculated as in Ref. [19].

Using this analysis we can calculate the function $I_m(Q_g,H,T)$ in which enters the unknown scale parameter $Y''(0)/Y'(0)^{3/2}$ and two adjustable parameters: (i) the parameter ρ of the reduction of I_0 due to penetration of magnetic field in the junctions [14] defined by $I_0(H)=I_0(1-\rho H^2)$ in the low field limit of relevance here and (ii) the critical field H_c such that $D(0,H>H_c)=0$, which corresponds to the field at which $I_m(Q_g)$ becomes e periodic at $T=0$. In Fig. 2(b), we plot $I_m(Q_g,H,T=65\ \mathrm{mK})/I_{m0}$ where $I_{m0}=I_m(Q_g=e,\ H=0,\ T=65\ \mathrm{mK})$ using the best fit values $\rho=18.5T^{-2}$ and $H_c=0.20$ T which are consistent with the junction geometry and with a previous measurement of D [19], respectively. These values are also used in the other comparisons described below. A close agreement with the experimental results is obtained. The validity of our model can be checked further on the temperature dependence of the I_m vs Q_g data shown in Fig. 4 taken for the intermediate field $H=0.11$ T. Experiments at higher temperatures agree less closely with theory, the relative amplitude of the peaks being greater in experiment than in theory. We believe this is due to the neglect of the departure of I_s from I_m induced by thermal fluctuations in the phase diffusion state. However, the nonmonotonous behavior of the $Q_g=e$ switching

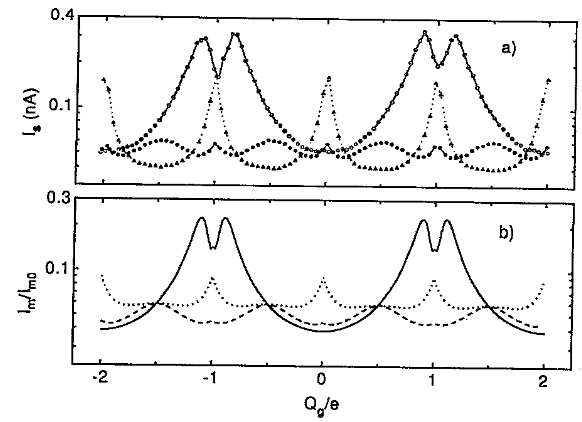

FIG. 4. (a) Switching current as a function of gate charge, at $H=0.11$ T and for several values of the temperature T, showing the complex transition from $2e$ periodicity to e periodicity with the increase of T. Open dots: $T=65$ mK; solid dots: $T=203$ mK; triangles: $T=356$ mK. (b) Theoretical runaway current as a function of gate charge, for the same temperature values as in (a) (the full and dotted lines correspond to the lowest and highest temperatures, respectively).

current as a function of temperature is well captured by our model, as shown in Fig. 5 where we also plot the $Q_g=0$ switching current for comparison. Note that the recovery above 250 mK of e periodicity, due to the vanishing of the odd-even free energy difference, was also found in other experiments [5,10]. Our model predicts the detailed features of this recovery: The odd manifold contributes dominantly to the current at $Q_g=0$ and the even manifold contributes dominantly to the current at $Q_g=e$ but, at intermediate temperatures, the switching current is maximum at $Q_g \simeq e/2$ as in the high field limit of Ref. [9].

In conclusion, we have shown that in a Josephson system where the number of quasiparticles was controlled, experimental measurements of charging effects can be explained by a minimal model, in contrast with preceding experiments. As Fig. 4 exemplifies, the competition between the charging energy, the Josephson energy, and the odd-even free energy difference produces a complex behavior of the supercurrent as a function of the gate charge, magnetic field, and temperature. This intrinsic complexity, together with the difficulties associated with the control of out-of-equilibrium quasiparticles, probably explains why the data in the superconducting state have always been found harder to interpret than in the normal state.

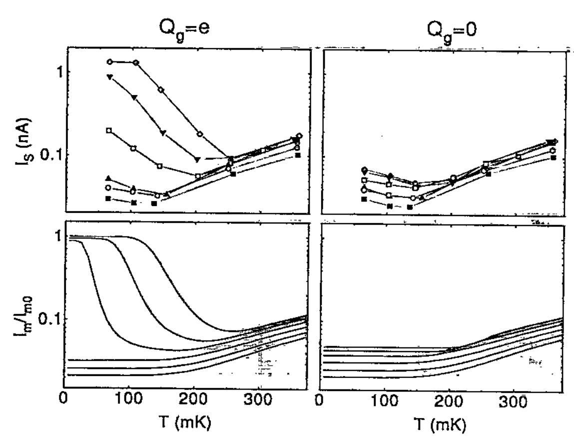

FIG. 5. Top panels: Experimental switching current as a function of temperature for $Q_g=0$ and $Q_g=e$. Top to bottom, same field values as in Fig. 2. Bottom panels: Theoretical runaway current for the same conditions as in top panels. The theory curves reproduce the strongly nonmonotonous temperature dependence found in the experiment at $Q_g=e$ for low fields.

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