Direct Measurement of the Josephson Supercurrent in an Ultrasmall Josephson Junction

A. Steinbach,[1] P. Joyez,[1] A. Cottet,[1] D. Esteve,[1] M. H. Devoret,[1] M. E. Huber,[2] and John M. Martinis[3]

[1]*Service de Physique de l'Etat Condensé, CEA-Saclay, F-91191 Gif-sur-Yvette, France*

[2]*Department of Physics, University of Colorado at Denver, Denver, Colorado 80217*

[3]*National Institute of Standards and Technology, Boulder, Colorado 80303*

(Received 10 April 2001; published 10 September 2001)

We have measured the supercurrent flowing through a nonhysteretic, ultrasmall, voltage-biased Josephson junction. In contrast with experiments performed so far on hysteretic Josephson junctions, we find a supercurrent peak whose maximum $I_{s\,\max}$ increases as the temperature T decreases. The asymptotic $T = 0$ value of $I_{s\,\max}$ agrees with the junction Ambegaokar-Baratoff critical current, as predicted by theory.

DOI: 10.1103/PhysRevLett.87.137003 PACS numbers: 74.50.+r, 73.23.–b, 73.40.Gk

A Josephson tunnel junction between two superconducting electrodes is a basic quantum nonlinear system [1,2]. For excitation energies much smaller than the superconducting gap Δ, it can be modeled by the Josephson Hamiltonian $\hat{H} = -E_J \cos\hat{\delta}$ where $\hat{\delta}$ is the gauge-invariant phase-difference operator, a purely electrodynamic quantity which is $2e/\hbar$ times the space and time integral of the electric field across the junction [2]. The Josephson energy E_J is a macroscopic parameter, which, for BCS superconductors at temperatures $T \ll \Delta/k_B$ and for sufficiently opaque junctions, depends only on the junction tunnel resistance R_t and Δ through $E_J = \frac{h}{8e^2}\Delta/R_t$ [3]. The supercurrent flowing through the junction is given by the Josephson relation $I_S = (2e/\hbar)\langle\partial\hat{H}/\partial\hat{\delta}\rangle = (2eE_J/\hbar)\langle\sin\hat{\delta}\rangle$, the average $\langle\cdots\rangle$ being performed on the degrees of freedom of the electrodynamic environment of the junction. Thus, the highest supercurrent that the junction can sustain is given by the so-called Ambekaokar-Baratoff critical current $I_0 = \frac{\pi}{2e}\Delta/R_t$ corresponding to an environment for which $\langle\sin\hat{\delta}\rangle = 1$. This critical current is easily observed for junctions with a small Coulomb energy $E_C = 2e^2/C_0 \ll E_J$ where C_0 is the intrinsic capacitance of the junction [4]. For these junctions, the phase behaves as a good quantum number δ which can be driven to the critical value $\delta = \pi/2$. However, for the so-called "ultrasmall" junctions characterized by $E_C \gtrsim E_J$, which are considered for applications in quantum information processing [5], the highest supercurrent has always been found experimentally well below the expected value I_0 [6–9]. Several untested hypotheses have been formulated to explain these results. The average $\langle\sin\hat{\delta}\rangle$ may not reach the value 1 because of uncontrolled quantum or thermal fluctuations. Failure of the Josephson Hamiltonian model for ultrasmall junctions could also explain the results, even if it is not directly expected from theory. Note that, experimentally, one cannot simply shunt the two leads of the junction by a small superconducting inductance to impose the phase difference, since it then becomes impossible to check the junction parameters by measuring its quasiparticle current.

The aim of the experiment reported in this Letter was to test the validity of the Josephson Hamiltonian model and the Josephson relation for an ultrasmall junction embedded in a controlled environment which should suppress phase fluctuations.

The principle of our experiment is shown schematically in Fig. 1a: an ultrasmall Josephson junction with critical current I_0 and intrinsic capacitance C_0 is biased by a circuit equivalent to a capacitor C_B in parallel with an ideal voltage source V_B in series with a resistance R_B. This bias circuit is also equivalent to a current source $I_B = V_B/R_B$ in parallel with R_B and C_B (Thévenin theorem). The average current I through the junction is measured by a current meter in series with the junction. The impedance of the meter is made negligible in comparison with the impedance of the bias circuit. The system is analogous to a damped quantum particle with mass $C(\hbar/2e)^2$, where $C = C_0 + C_B$, placed in a tilted washboard potential $U(\hat{\delta}) = -E_J\cos\hat{\delta} - I_B(\hbar/2e)\hat{\delta}$. The damping due to the resistance R_B, assumed to be in thermal equilibrium at temperature T, manifests itself also as a fluctuating force acting on the particle [2]. We consider only the overdamped regime $R_B \ll \sqrt{\hbar/(2eCI_0)}$, for which the particle mass can be neglected. In this regime, all quantum fluctuations of the phase are suppressed, provided that $R_B \ll h/(2e)^2$ [10]. Previous experiments have tried to implement this idealized circuit and determine the supercurrent maximum $I_{S\,\max} = \max[I_S(V_B)]$ which should tend to I_0 as $T \to 0$. However, instead of the pure R_B and C_B combination, all of them had a strongly frequency-dependent and often ill-characterized impedance $Z(\omega)$. This difficulty arose because the measuring setups involved high input impedance field-effect transistor amplifiers and the dc value of Z had to be made large in order to resolve the contribution of the junction quasiparticle current to the $I(V)$ characteristics. In practice, the condition for hysteresis $Z(\omega = 0) \gg \sqrt{\hbar/(2eC_0I_0)}$ was inevitable. The zero-voltage state, in which the supercurrent is measurable, was therefore metastable and was switching to the nonzero voltage state at $I_B = I_{\rm sw} < I_{S\,\max}$ [11]. This switching

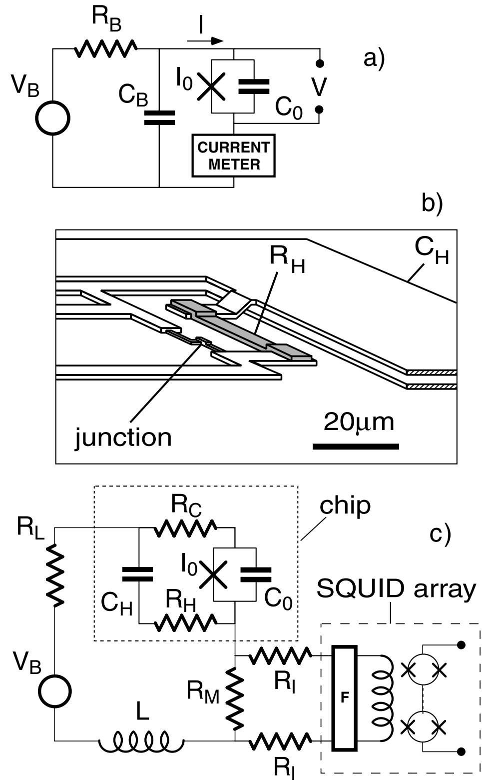

FIG. 1. (a) Idealized circuit for measurement of supercurrent of Josephson junction. The junction consists of a Josephson element (cross) in parallel with a capacitor. (b) On-chip high-frequency circuitry contributing to electrodynamic damping of Josephson junction. For clarity, metallic thin films only are represented. (c) Measurement setup schematics including both on-chip circuitry (box in dotted line) and off-chip circuitry. A known fraction of the current through the junction is coupled to a SQUID array (box in dashed line). The backaction noise of the SQUID array is attenuated by filter F.

transition is affected by thermal and quantum fluctuations, and I_{sw} is characterized by a probability distribution. The average $\bar{I}_{sw}$ obtained so far for ultrasmall junctions showed various degrees of reduction compared to I_0 [6], ranging from around $10^{-3}I_0$ to $0.65I_0$, this largest value having been obtained using an on-chip impedance [9].

In the present experiment, we have circumvented these problems and measured the current through a junction in the nonhysteretic regime using as the current meter a recently developed SQUID series array with 100 dc SQUIDs [12]. The impedance of the current-measuring circuit is so low that we can afford to overdamp conservatively the junction at all frequencies, thereby ensuring that the measurement finds the junction in a fully stable state with a controlled absence of quantum fluctuations. The total impedance seen by the junction is equivalent to a pure resistor $R_B = 24 \pm 1\ \Omega$ in parallel with a reactive element which behaves as a capacitor $C_B = 200 \pm 20$ fF above a few tens of MHz. In conventional measuring setups, such heavy damping would make the junction current hardly distinguishable from the current through the shunt resistor.

The actual measurement setup implementing the idealized Fig. 1a circuit is shown in Figs. 1b and 1c. It has been designed to minimize deviations of the environmental impedance from the ideal limit $Z(\omega)^{-1} = R_B^{-1} + jC_B\omega$ while maintaining dissipative elements in thermal equilibrium at a controlled temperature T, as well as imposing an accurate bias current. In order to meet, in a frequency range spanning 10 orders of magnitude, these conflicting requirements, we have engineered an environment which consists of both microscopic on-chip (Fig. 1b) and macroscopic off-chip components (see outside of dotted line box in Fig. 1c). The on-chip components contribute mainly to the high-frequency values of the environmental impedance, which include the junction bare plasma frequency $\sqrt{2eI_0/(\hbar C_0)}$ in the tens of GHz range, while the off-chip components contribute mostly to the low-frequency values, which includes possible Josephson resonances in the hundreds of MHz range, as well as the measurement frequencies below a few kHz. The on-chip circuitry consists of a resistance $R_H = 11.8\ \Omega$ in series with a capacitance $C_H \simeq 100$ pF, which were fabricated using a five-layer optical lithography process. The resistor R_H was made from 150 nm thick AuPd with a width and length of approximately 5 μm by 25 μm. This small resistor was in good electrical contact with a large Au pad that served as a thermal reservoir. A single Al-AlO$_x$-Al Josephson junction was fabricated by e-beam lithography and double angle shadow mask evaporation [13]. We estimate the capacitance $C_0 = 1$ fF from the junction area. The contact resistance resulting from the junction fabrication process was $R_C = 12.1\ \Omega$. The off-chip components $R_M = 1.67\ \Omega$, $R_I = 10\ \Omega$, and $R_L = 10.1\ \Omega$ were surface mounted resistors for microwave circuits placed within 5 mm of the junction to minimize the stray inductance $L \simeq 4$ nH of the connection between the off-chip and on-chip circuitry. The role of R_M and the two R_I's is to provide a current divider for minimizing the backaction of the Josephson oscillations inside the SQUID array on the measured junction. A microwave copper-powder filter [14] placed in series with the R_I resistors provides further attenuation at high frequency. Only about 8% of the junction supercurrent was thus coupled to the SQUID array. The biasing circuitry at high temperature was connected to the resistor R_L through coaxial lines filtered by a combination of copper-powder filters and miniature cryogenic filters [15], and its action is equivalent to a voltage source V_B in series with R_L. The sum of R_L, R_M, and R_C determines the dc value of the environmental impedance, while the sum of R_H and R_C determines its high-frequency value (inductor L blocks high-frequency currents). The sample and the low temperature bias circuitry were thermally anchored inside a copper box bolted to the mixing chamber of a dilution refrigerator.

The junction $I(V)$ characteristic is shown in Fig. 2a for $T = 34$ mK. The superconducting gap is directly measured to be $\Delta = 200 \pm 2\ \mu$V and the junction normal

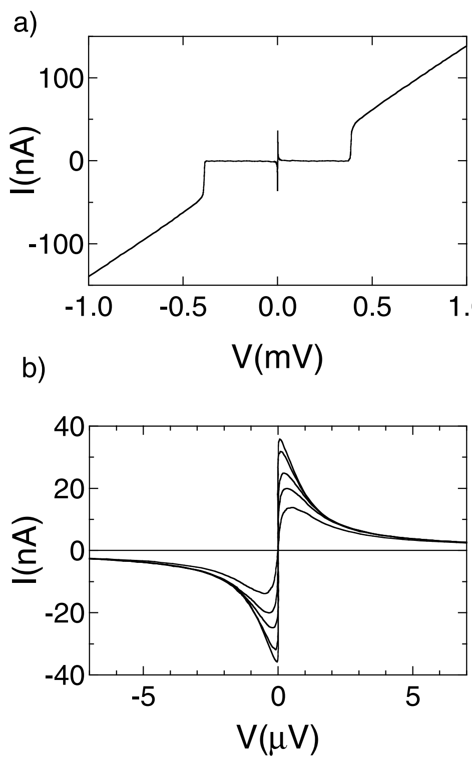

FIG. 2. (a) Large-scale $I(V)$ characteristic of the ultrasmall Josephson junction of Figs. 1b and 1c at $T = 34$ mK. (b) Josephson supercurrent peak shown on an expanded voltage scale, at different temperatures. From top to bottom: $T = 34, 98, 245, 400, 622$ mK, respectively.

state resistance approaches the asymptotic value $R_N = 6.99\ \mathrm{k}\Omega$ at voltages many times $2\Delta/e$. The critical current is then calculated to be $I_0 = 44.9 \pm 0.5$ nA. On first inspection, the $I(V)$ of Fig. 2a appears conventional, with the Josephson current manifesting itself as a vertical line at zero voltage. However, because of the low impedance of our biasing circuit, there is no hysteresis and all points on the $I(V)$ characteristic are stable, in contrast with the usual current-bias ramp method giving access only to the positive differential conductance part of the $I(V)$ in the best cases. Thus, it is worth stressing that in our experiment, a *supercurrent peak,* as opposed to a supercurrent branch, is measured for the first time. The detailed structure of the supercurrent peak is shown for several temperatures in Fig. 2b. The higher temperature data show a finite slope around zero bias which is due to phase diffusion in the tilted washboard potential [16]. The full $I(V)$ characteristic in the pure Ohmic damping case was first calculated by Ivanchenko and Zil'berman [17]:

$$I(V_B) = I_0\, \mathrm{Im}\left[\frac{I_{1-2i\beta eV_B/\hbar R_B}(\beta E_J)}{I_{-2i\beta eV_B/\hbar R_B}(\beta E_J)}\right], \tag{1}$$

where $I_\nu(z)$ is the modified Bessel function, $\beta = 1/k_B T$, $V_B = V + R_B I$. The more general approach [18], developed to solve the steady-state Fokker-Planck equation for the phase distribution in a tilted washboardlike potential, has been proved to yield equivalent results [19]. The expression (1) predicts a supercurrent peak with a maximum which tends to I_0 in the zero temperature limit. A detailed comparison between the $I(V)$ characteristics measured at three temperatures and the theoretical predictions are shown in Fig. 3 with no adjustable parameters. The close agreement between theory and experiment around the peak maximum shows that the temperature of the electromagnetic environment which drives the phase dynamics is indeed equal to the experimental refrigerator temperature. The agreement over the whole voltage range, without any spurious resonances, confirms that the impedance of the junction environment is indeed almost constant over the corresponding range of Josephson frequencies. As a check, we have simulated the classical dynamics of a small junction for the exact circuit of Fig. 1b, including the effect of thermal fluctuations. The theoretical $I(V)$ curves so obtained are negligibly different from those obtained with expression (1) with $R_B = 24\ \Omega$, which indicates that our experiment implements the ideal bias case satisfactorily. As a further check of the influence of the off-chip bias circuitry on the $I(V)$, we have increased L to a value of order 100 nH and observed that the $I(V)$ then developed two metastable branches predicted by our numerical simulations and corresponding to chaotic Josephson oscillations in the hundreds of MHz range [20].

In Fig. 4, we compare the measured supercurrent peak height $I_{S\,\max}$ with the values predicted from (1) over a large temperature range [21]. The peak height $I_{S\,\max}$ increases as the temperature is lowered down to 26 mK, in agreement with the classical theory. However, the agreement between theory and experiment below 200 mK was attained only after the filtering at the input of the SQUID array, as described in Fig. 1c, had been installed. This indicates the magnitude of the backaction noise produced by this type of amplifier. We do not have a fully convincing explanation for the deviations between experiment and theory at

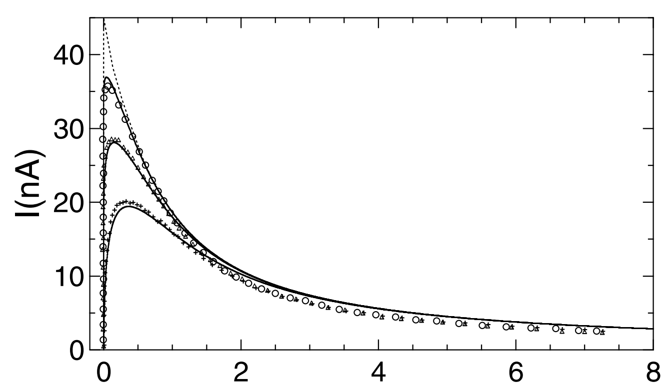

FIG. 3. Comparison between the $I(V)$ characteristics measured at different temperatures (symbols) and the calculated ones (full lines) using Eq. (1) and $I_0 = 44.9$ nA and $R = 24\ \Omega$. From top to bottom: $T = 34$, 157, and 400 mK, respectively. Dashed line represents the $I(V)$ predicted at $T = 0$.

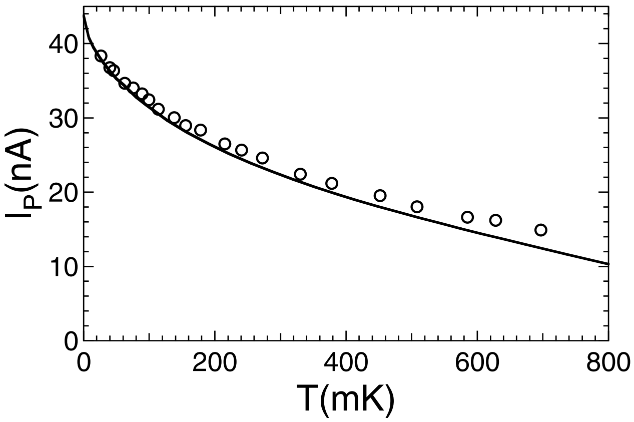

FIG. 4. Comparison between the measured temperature dependence of the maximum supercurrent (open circles) and the predicted one (full line).

the highest temperatures, but the contribution of the quasiparticle current to the damping of the junction, which we neglect in our analysis, may play a role in this regime.

Our experiment thus provides strong experimental evidence that the commonly observed reduction of the maximum supercurrent in an ultrasmall junction is not an intrinsic junction property, but is due to its electrodynamic environment. When the environment is engineered to place the junction in the overdamped regime in a controlled manner, the Ambegaokar-Baratoff critical current I_0 can be reached at low temperature, thereby showing the validity of the Josephson Hamiltonian for ultrasmall junctions. A control of the environment impedance similar to that of our experiment, but with higher resistances, would allow the observation of the strong reduction of the maximum supercurrent by quantum fluctuations, which has been recently predicted by Ingold and Grabert [22] in the case of a resistive environment with $R \sim R_Q$.

Our work also provides ground for the application of the single Cooper pair transistor (SCPT) [7] to electrometry. This device consists of two small Josephson junctions in series. At low temperature, it is equivalent to a single junction whose Josephson energy is modulated with a $2e$ period by the gate charge coupled to the island formed between the two junctions. This modulation can be exploited for low-noise-temperature electrometry [23], provided that the device supercurrent is measured like in the present experiment. The SCPT could operate at high frequencies since intrinsic bandwidths up to 120 and 250 MHz have been demonstrated for arrays with 100 and 30 SQUIDs, respectively [24]. Further work is needed to know how this new type of electrometer competes in fast electrometry with the recently developed RF-SET [25].

We gratefully acknowledge discussions with J. Imry, G.-L. Ingold, and H. Grabert. This work was partly supported by the European Union through Contract No. IST-10673 SQUBIT and by the Bureau National de la Métrologie.

[1] B. D. Josephson, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969).
[2] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
[3] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963).
[4] A. Barone and G. Paternò, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[5] M. F. Bocko, A. M. Herr, and M. F. Feldman, IEEE Trans. Appl. Supercond. **7**, 3638 (1997); J. E. Mooij *et al.,* Science **285**, 1036 (1999); D. V. Averin, Solid State Commun. **105**, 659 (1998); Yu. Makhlin, G. Schoen, and A. Shnirman, Nature (London) **398**, 305 (1999); Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[6] M. Tinkham, in *Single Charge Tunneling,* edited by H. Grabert and M. Devoret (Plenum Press, New York, 1992).
[7] P. Joyez *et al.,* Phys. Rev. Lett. **72**, 2458 (1994).
[8] W. J. Elion *et al.,* Nature (London) **371**, 594 (1994).
[9] D. Vion *et al.,* Phys. Rev. Lett. **77**, 3435 (1996); P. Joyez *et al.,* J. Supercond. **12**, 757 (1999).
[10] H. Grabert, G.-L. Ingold, and B. Paul, [Europhys. Lett. **44**, 360 (1998)], have established that for the circuit shown in Fig. 1a, and in the limit $R_B \ll R_Q = h/(2e)^2$, the junction dynamics is the same as if the phase was a classical variable, but with a renormalized E_J depending on both R_B and $C = C_B + C_0$. According to this theory, the quantum corrections to the bare E_J are negligible for our experiment (note that both R_B and C_B contribute to driving the junction into the classical phase regime).
[11] T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B **9**, 4760 (1974).
[12] R. P. Welty and J. M. Martinis, IEEE Trans. Magn. **27**, 2924 (1991); M. E. Huber *et al.,* Appl. Supercond. **5**, 425 (1998).
[13] G. J. Dolan and J. H. Dunsmuir, Physica (Amsterdam) **152B**, 7 (1988).
[14] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).
[15] D. Vion *et al.,* J. Appl. Phys. **77**, 2519 (1995).
[16] J. M. Martinis and R. L. Kautz, Phys. Rev. Lett. **63**, 1507 (1989); R. L. Kautz and J. M. Martinis, Phys. Rev. B **42**, 9903 (1990).
[17] Yu. M. Ivanchenko and L. A. Zil'berman, Sov. Phys. JETP **28**, 1272 (1969).
[18] V. Ambegaokar and B. I. Halperin, Phys. Rev. Lett. **22**, 1364 (1969).
[19] W. T. Coffey, Yu. P. Kalmykov, and J. T. Waldron, *The Langevin Equation* (World Scientific, Singapore, 1996).
[20] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits* (Gordon and Beach, New York, 1986), pp. 177–180.
[21] Small corrections to the Josephson relation have been applied for temperatures above 600 mK following Ref. [3].
[22] G.-L. Ingold and H. Grabert, Phys. Rev. Lett. **83**, 3721 (1999).
[23] A. B. Zorin *et al.,* J. Supercond. **12**, 747 (1999).
[24] M. Huber *et al.,* IEEE Trans. Appl. Supercond. **11**, 1251 (2001).
[25] R. J. Schoelkopf *et al.,* Science **280**, 1238 (1998); A. Aassime, G. Johansson, G. Wendin, R. J. Schoelkopf, and P. Delsing, Phys. Rev. Lett. **86**, 3376 (2001).