MEASURING QUANTRONIUM QUBIT WITH CAVITY BIFURCATION AMPLIFIER

Michael Metcalfe

E. Boaknin, V. Manucharyan, R. Vijay, C. Rigetti, I. Siddiqi, L. Frunzio and M. H. Devoret
(Qlab, Department of Applied Physics, Yale University)

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Monday evening seminar
September 11th 2006
1. Introduction: - Quantum computing
   - Implementations
   - Quantum circuits
   - Quantronium

2. Cavity bifurcation amplifier (CBA):
   - Non-linear readout
   - Implementations
   - Bistability

3. Quantronium with CBA:
   - Spectroscopy/Gate modulations
   - Rabi/Ramsey oscillations

4. Outlook:
   - Coupled qubits (See Chad’s talk)
   - Multiplexed C.B.A’s
Quantum computation:
Unitary evolution of initially prepared n-qubit state and its subsequent measurement

Qubit:
Just a 2-state system (effective spin – ½)

\[ |\varphi\rangle = \alpha |0\rangle + \beta |1\rangle \]

Isolate two energy levels
QUANTUM PARALLELISM

N-qubit state:

\[ |\psi\rangle = \sum_{k=0}^{2^N-1} C_k |k\rangle \]

Unitary operation, U:

\[ U|\psi\rangle = \sum_{k=0}^{2^N-1} C_k U |k\rangle \]

2^N operations in parallel!!
QUBIT REALIZATIONS

Trapped ions

- Long qubit coherence
- Low environmental coupling
- Scalability difficult

NMR

- Chloroform molecule

Semiconductor dots

- Shorter qubit coherence
- Large environmental coupling
- Scalability easier

Superconducting circuits

- Electrical access
QUANTUM CIRCUITS

\[ H = \frac{\phi^2}{2L} + \frac{q^2}{2C} \]

\[ [\phi, q] = i\hbar \]

\[ \Delta E = \hbar \omega \leq k_B T \]

Microfabrication and cryogenics

\( \omega \leq 1\text{GHz} \)
\( T \leq 10\text{mK} \)

Equal level spacing

Need non-linear element
NON-LINEAR ELEMENT: JOSEPHSON JUNCTION

Josephson current:

\[ i(t) = -I_0 \sin \left( \frac{2e}{\hbar} \Phi_J(t) \right) \]

Inductance:

\[ L = \frac{1}{\frac{di}{d\Phi}} = \frac{\hbar}{2eI_0 \cos(\Delta\phi)} \]

Non-Linear Inductor!
\[ \hat{H}(N_g) = E_{CP} \left( \hat{N} - N_g \right)^2 - E_J \cos(\hat{\theta}) \]

\[ E_{CP} = \frac{(2e)^2}{2(C_g + C_J)} \]

\[ N_g = \frac{C_g V_g}{2e} \]

\[ [\hat{\theta}, \hat{N}] = i \]

\[ E/E_{CP} \]

\[ |0\rangle \]

\[ |1\rangle \]

\[ |2\rangle \]
SPLIT COOPER PAIR BOX

\[
\hat{H}(N_g) = E_{CP} \left( \frac{1}{i \frac{\partial}{\partial \theta}} - N_g \right)^2 - E_J(\delta) \cos\left( \hat{\theta}(\delta) \right)
\]

\[
E_J \ll E_{CP} \quad \Rightarrow \quad H_{CPB} = h_z(\delta) \sigma_z + h_x(N_g) \sigma_x
\]

Artificial 2-level atom
ENGLISH LEVELS

Eigenstates, $E_k$: Mathieu Functions

Sweet spot:
\[ \frac{\partial E_k}{\partial \delta} = \frac{\partial E_k}{\partial N_g} = 0 \]

Average loop current:
\[ i_k(N_g, \delta) = \frac{1}{\phi_0} \frac{\partial E_k(N_g, \delta)}{\partial \delta} \]

With inductance:
\[ L(N_g, \delta) = \left[ \frac{1}{\phi_0} \left( \frac{\partial I}{\partial \delta} \right) \right]^{-1} = \left[ \frac{1}{\phi_0^2} \left( \frac{\partial^2 E_k(N_g, \delta)}{\partial \delta^2} \right) \right]^{-1} \]
Non-linear LC oscillator

\[ V_{\text{drive}} \]

\[ R \quad C \quad L \quad L_J \]

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\[ |V_{\text{out}}| \]

\[ \omega \]

\[ V_d \]
NON-LINEAR READOUT

Non-linear LC oscillator

Quantum Qubit

Energy levels

Mapping
IMPLEMENTATIONS

Quantronium qubit with Josephson bifurcation amplifier (JBA)

Quantronium qubit with cavity bifurcation amplifier (CBA)

(I. Siddiqi et. al, PRB 2006)
1) Precisely control non-linearity
2) Engineer freq, Q
3) No dissipation on-chip
MICROWAVE RESPONSE

Bistable states

1.8 GHz

9.8 GHz
OBSERVATION OF BISTABILITY
WHY: MULTIPLEXED QUBITS

- Coupling line
- Qubit in resonator
- Multiplexed readout and gate lines
C.B.A. WITH QUANTRONIUM

SEM image of Quantronium in CBA

10 µm

Island + 2 junctions

Readout junction

CPW ground

CPW ground
GATE MODULATIONS

Fit is with $E_{cp} = 17.0\text{GHz}$ and $E_{J0} = 15.1\text{GHz}$

Readout in linear regime

Move to bistable regime:

More gain
SPECTROSCOPY

Fit: $E_{cp} = 17.0$GHz
$E_{J0} = 15.1$GHz
RABI OSCILLATIONS

qubit pulse

\[ \tau \]

Readout

A

\[ A, \text{ (Volts)} \]

\[ A^2, \text{ (dBm)} \]

Y-rotation

\[ |1\rangle \]

\[ |0\rangle \]

Switching Probability (shifted)

Time (\(\mu s\), \(\tau\))

Rabi frequency (MHz)

A, (Volts)

A, (Cooper pairs)
BEST RABI OSCILLATIONS

Switching probability

Time (μs)

data
fit
RELAXATION TIME, $T_1$

$\pi$ pulse

$\tau_{\omega}$

Readout

$T_1 = 1.4 \mu s \leftrightarrow 1.8 \mu s$

Switching Probability

Time, $\tau_{\omega}$ (µS)

Data

Fit

$|1\rangle$

$|0\rangle$
RAMSEY FRINGES

$\pi/2$ pulse $\tau$ $\pi/2$ pulse

Readout

$|1\rangle$

$|0\rangle$

$200\text{ns} \leq T_2 \leq 800\text{ns}$

$Q_\phi = 2\pi v_{01} T_2 = 2.10^4 \leftrightarrow 7.10^4$
MEASURED CONTRAST

$\pi$ pulse

Readout

Readout relaxation → Reduced contrast

Switching probability

Readout pulse amplitude (V)
TOMOGRAPHY

Preparation pulse

Tom. pulse

Readout

Initial states

|0⟩

|0⟩+|1⟩/√2

|1⟩

|0⟩−|1⟩/√2

|0⟩
NOW: SLOTLINE NON-LINEAR RESONATOR

Optical image of device

SEM image of qubit

Advantages: 1. Completely fabricated using e-beam lithography
2. Gate and readout lines separated
3. Excite $\pm$ mode in slotline for readout
NEAR FUTURE: COUPLED QUBITS

See Chad’s MES upcoming.....

Optical image of coupled qubit with slotline readout
LONGER TERM: MANY COUPLED MULTIPLEXED QUIBTS

Multiplexed resonators

3mm

10mm

Nb

preliminary data: amplitude response

Output Power (dB)

Frequency (GHz)

Input Power (dBm)
CONCLUSION

• Developed fast, non-dissipative RF readout
• Single shot qubit readout feasible
• Scalable architecture
• Coupled qubit experiments in progress…
PREDICTED CONTRAST

\[ \Delta I_0 = 0.4 \text{nA} \]
\[ I_0 = 0.3 \mu \text{A} \]

\[ \Delta \omega \]

\[ 2 \text{GHz} \quad 12 \text{mK} \]

<table>
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<tr>
<th>Freq</th>
<th>( \Delta \omega )</th>
<th>( \Delta I_0^{\text{meas}} )</th>
<th>( \Delta I_0^{\text{theory}} )</th>
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<tr>
<td>2GHz</td>
<td>1.4 MHz</td>
<td>0.4 nA</td>
<td>0.5 nA</td>
</tr>
<tr>
<td>10GHz</td>
<td>20 MHz</td>
<td>6.4 nA</td>
<td>5 nA</td>
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Single shot possible
RAMSEY TOMOGRAPHY

Map of state during Ramsey experiment (c.f. Martinis, PRL)
Slides after this are additional
Joeysphon Junction:

Charge tunneling through JJ:

\[ Q = 2eN \]

Flux associated with SC phase diff across JJ:

\[ \Phi = \varphi_0 \theta \]

\[ [\theta, N] = i \]

Hamilton of JJ:

\[ H = E_{CJ} \left( N - \frac{Q}{2e} \right)^2 - E_J \cos \theta \]

where

\[ E_{CJ} = \frac{(2e)^2}{2C_J} \]

Cooper Pair box:

\[ \tilde{H}(N_g) = E_c \left( \frac{1}{i} \frac{\partial}{\partial \theta} - N_g \right)^2 - E_J \cos(\tilde{\theta}) \]

\[ N = \frac{1}{i} \frac{\partial}{\partial \theta} \]

\[ E_J << E_{CP} \]

\[ H_{CPB} = -E_z \left( \sigma_z + X_{cont} \sigma_x \right) \]

where

\[ E_z = E_J / 2 \]

\[ X_{cont} = 2E_{CP} / E_J \left( \frac{\sqrt{2}}{2} - N_g \right) \]

\[ E_J \leftrightarrow \text{Zeeman field} \]

\[ E_{CP} \leftrightarrow \text{transverse field} \]

\[ X_{cont} = 0 \Rightarrow \text{Sweet spot} \]

Loop current/Inductances:

\[ \tilde{i}(N_g, \delta) = -2e \frac{dK}{dt} \]

where number of cooper pairs tunnelling onto island

\[ \hat{K} = \left( \hat{N}_1 + \hat{N}_2 \right) / 2 \]

\[ \tilde{i}(N_g, \delta) = -\frac{1}{i\varphi_0} \left[ \hat{K}, \hat{H} \right] = \frac{1}{\varphi_0} \frac{\partial \hat{H}}{\partial \delta} \]

Hence the average loop current is:

\[ i_k(N_g, \delta) = \frac{1}{\varphi_0} \frac{\partial E_k(N_g, \delta)}{\partial \delta} \]

with inductance:

\[ L(N_g, \delta) = \left[ \frac{1}{\varphi_0} \left( \frac{\partial I}{\partial \delta} \right) \right]^{-1} = \frac{1}{\varphi_0^2} \left( \frac{\partial^2 E_k(N_g, \delta)}{\partial \delta^2} \right) \]
NON-LINEAR OSCILLATOR

\[ L_t \ddot{q} + R \dot{q} + \frac{q}{C} + L_J \ddot{q} \left( \frac{1}{\sqrt{1 - (\dot{q} / I_0)^2}} - 1 \right) = V_d \cos \omega t + V_N \]

Non-Linear resonance
Readout scheme

\[ I_0'' < I_0' \]

Bias point

\[ V_{\text{drive}} \]

\[ |V_{\text{out}}| \]
Previous work

Quantronium qubit with JBA

I. Siddiqi et. al, PRB 2006

CQED

A. Wallraff et. al, Nature 2004

Flux qubit with linear resonator and JBA

A. Lupascu et. al cond-mat/0601634
Experimental observation of two oscillator states
Microwave response

Bistable states

1.8 GHz

9.8 GHz
Bistability
Readout scheme

![Diagram showing the readout scheme with states 0 and 1, oscillation amplitude vs. power in, and bias point.](Image)
- Complicated fabrication
- Same physics
TIME RESOLVED MEASUREMENT

Diagram showing a time-resolved measurement setup with various components such as sources, step attenuators, cold Russian amplifiers, and filters.
ESCAPE RATES

For: \( \omega_a \not\in \Gamma \)

Thermally activated escape:

\[
\gamma = \frac{\omega_a}{2\pi} \exp\left(-\frac{\Delta U}{k_B T_{\text{esc}}}\right)
\]

Where:

\[
\omega_a = \frac{2}{3\sqrt{3}} \Gamma \Omega^2 |1 - \beta / \beta_b|^{1/2} \quad \Delta U = \frac{64}{9\sqrt{3}} E J \left(\frac{L_i}{L_j}\right)^2 \frac{\Omega}{Q} |1 - \beta / \beta_b|^{3/2}
\]

Plot: \( \left(\ln\left(\frac{\omega_a}{2\pi\gamma}\right)\right)^{2/3} \) vs \( \beta / \beta_b \) \( \rightarrow T_{\text{esc}} \)
ESCAPE RATES
METHOD

\[ \gamma(V) = \frac{1}{\Delta V} \frac{dV}{dt} \ln \left( \frac{\sum_{v \geq V} P(v)}{\sum_{v \geq V + \Delta V} P(v)} \right) \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{esr_method.png}
\end{figure}
ESCAPE RATES

\[ \left( \frac{\omega_a}{2 \pi \gamma} \right)^{2/3} \]

Intercept: \( V_b \)
Slope: \( T_{\text{esc}} \)

\[ \frac{U_0}{K_B T} = 420 \]
\[ U_0 = 220K \]
\[ T_{\text{esc}} = 520\text{mK} \]
\[ T_{\text{physical}} = 260\text{mK} \]

Useful tool
2D-HISTOGRAMS

No latching

Real (arb)

Imaginary (arb)

latching

Real (arb)

Imaginary (arb)

$\tau_m - 2\mu s$

$\tau_m - 10\mu s$

2µs 2µs 11µs 10ns

2µs 3µs 10ns
CAVITY BIFURCATION AMPLIFIER

Parameters:
- $Z_0 = 50\Omega$;
- $\omega_0 \sim (1.8\text{Ghz})2\pi$;
- $C_{\text{OUT}} \sim 50fF$;
- $Q \sim 2000$

\[
L_t \ddot{q} + R_{\text{eff}} \dot{q} + \frac{q}{C_{\text{eff}}} + L_J \dot{q} \left( \frac{1}{\sqrt{1 - (\dot{q} / I_0)^2}} - 1 \right) = V_d \cos \omega t + V_N
\]
THE JOSEPHSON TUNNEL JUNCTION: AN ATOM-LIKE CIRCUIT ELEMENT TO WHICH YOU CAN ATTACH WIRES ...

\[ [\hat{\theta}, \hat{N}] = i \]  

NON-LINEAR INDUCTOR
JOSEPHSON JUNCTION

Zero voltage supercurrent:

\[ I_s = I_c \sin(\Delta \varphi) \]

\[ \phi = \frac{\Phi_J(t)}{\Phi_0} \]

\[ \Phi_0 = \frac{\hbar}{2e} \]

Applied voltage:

\[ \frac{d(\Delta \varphi)}{dt} = \frac{2eV}{\hbar} \]

\[ \Phi_J = \int_{-\infty}^{t} \nu(t') dt' \]

→ alternating current

Free energy stored in junction:

\[ F = -E_J \cos(\Delta \varphi) + \text{const} \]

\[ E_J = \frac{\hbar I_c}{2e} \]

\[ L_J = \frac{\hbar}{2eI_0} \]

Non-Linear Inductor!
ENERGY LEVELS OF AN ISOLATED JUNCTION

HAMILTONIAN: \[ H = E_{cp} \left( \hat{N} - q / 2e \right)^2 - E_J \cos(\theta) \]

\[ Z >> R_Q = \frac{\hbar}{4e^2} \]

CHARGING ENERGY

COUPLING ENERGY, \( U_J \)

E/E_{CP}

BLOCH WAVEFUNCTIONS
SHADOW-MASK EVAPORATION

E-BEAM WRITE PATTERN WITH SEM

3.5µm OPTICAL IMAGE OF RESIST

PLEXIGLAS (PMMA)

COPOLYMER (MMA 8.5)

SILICON

SPIN ON RESIST

DICE WAFER INTO SMALL CHIPS

WRITE PATTERN WITH SEM

2" SI WAFER
SHADOW-MASK EVAPORATION

CROSS SECTION:

SEM IMAGE OF AL/AL$_2$O$_3$/AL JUNCTION

3.5µm

Oxidation: 15% O$_2$, 85% Ar, 1-10 Torr, 5-20 mins

2nd Al evaporation: 2A, 1:3, for ~40s

Lift off resist with unwanted Al with Acetone (60-90°C)

Develop in MIBK:IPA, 1:3, for ~40s and rinse in IPA

PMMA

MMA

Si

Undercut

Bridge
NON-LINEAR OSCILLATOR

\[ L_t \ddot{q} + R \dot{q} + \frac{q}{C} + L_J \dot{q} \left( \frac{1}{\sqrt{1 - (\dot{q} / I_0)^2}} - 1 \right) = V_d \cos \omega t + V_N \]

Non-Linear resonance